微積分レポート正解

レポート 1 解答　　17 点

問 1.

(1) $X \cup Y = \boxed{\{2, 3, 4, 7, 8\}}$

(2) $X \cap Y = \boxed{\{3, 7\}}$

(3) $\overline{X} = \boxed{\{1, 4, 5, 6, 8\}}$

問 2.

(1) $\boxed{1}$: $\boxed{\exists x \in X, \exists y \in Y : x + 4 \leq y}$

$\boxed{2}$: $\boxed{1}$

$\boxed{3}$: $\boxed{5}$

(2) $\boxed{4}$: $\boxed{\exists x \in X, \forall y \in Y : 6 - x \leq y}$

$\boxed{5}$: $\boxed{4}$

$\boxed{6}$: $\boxed{2}$

(3) $\boxed{7}$: $\boxed{\forall x \in X, \exists y \in Y : (x - 2)^2 < y \leq x + 1}$

$\boxed{8}$: $\boxed{2}$

$\boxed{9}$: $\boxed{5}$

レポート 2 解答

(1) (証明)　$\forall\epsilon > 0$ に対して、$\dfrac{a}{\epsilon} < N$ となる自然数 N を取る。

すると、$n > N$ ならば、$\left|\dfrac{a}{n}\right| < \dfrac{a}{N} < \epsilon$　　$\square$

(2) (証明)　$\forall\epsilon > 0$ に対して、$\dfrac{1}{\sqrt{\epsilon}} < N$ となる自然数 N を取る。

すると、$n > N$ ならば、$\left|\dfrac{1}{n^2}\right| < \dfrac{1}{N^2} < \epsilon$　　$\square$

(3) (証明)　$\forall\epsilon > 0$ に対して、$\dfrac{1+a+\epsilon}{\epsilon} < N$ となる自然数 N を取る。

すると、$n > N$ ならば、$\dfrac{1+a+\epsilon}{\epsilon} < n$

$1 + a + \epsilon < n\epsilon$
$1 + a < (n-1)\epsilon$

$\left|\dfrac{an+1}{n-1} - a\right| < \dfrac{1+a}{n-1} < \epsilon$　　$\square$

(4) (証明)　$\forall\epsilon > 0$ に対して、$\dfrac{5-3\epsilon}{9\epsilon} < N$ となる自然数 N を取る。

すると、$n > N$ ならば、$\dfrac{5-3\epsilon}{9\epsilon} < N < n$

$5 - 3\epsilon < 9n\epsilon$
$5 < (9n+3)\epsilon$

$\left|\dfrac{4n+3}{3n+1} - \dfrac{4}{3}\right| = \dfrac{5}{9n+3} < \epsilon$　　$\square$

レポート 3 解答　　9 点

(1) $y' = \boxed{2^x \log 2}$

(2) $y' = \boxed{\dfrac{1}{x \log 3}}$

(3) $y' = \boxed{2e^{2x+3}}$

(4) $y' = \dfrac{(x^2-2x+3)'}{x^2-2x+3} = \boxed{\dfrac{2x-2}{x^2-2x+3}}$

(5) $y' = (2-x)'e^{x^2} + (2-x)(e^{x^2})' = -e^{x^2} + (2-x)e^{x^2}(x^2)'$

$= -e^{x^2} + (2-x)e^{x^2}(2x) = \boxed{(-1+4x-2x^2)e^{x^2}}$

注)　$(fg)' = f'g + fg'$

(6) $y' = (3^x)' \log_2 x + 3^x (\log_2 x)' = \boxed{3^x \log 3 \log_2 x + \dfrac{3^x}{x \log 2}}$

(7) $y' = (\log|x-a| - \log|x+a|)' = \dfrac{1}{x-a} - \dfrac{1}{x+a} = \dfrac{(x+a)-(x-a)}{(x-a)(x+a)} = \boxed{\dfrac{2a}{x^2-a^2}}$

注)　$\log \dfrac{a}{b} = \log a - \log b$

(8) $y' = \dfrac{(x+\sqrt{x^2+A})'}{x+\sqrt{x^2+A}} = \dfrac{1+\frac{(x^2+A)'}{2\sqrt{x^2+A}}}{x+\sqrt{x^2+A}} = \dfrac{1+\frac{2x}{2\sqrt{x^2+A}}}{x+\sqrt{x^2+A}}$

$= \dfrac{\frac{\sqrt{x^2+A}+x}{\sqrt{x^2+A}}}{x+\sqrt{x^2+A}} = \dfrac{\sqrt{x^2+A}+x}{(x+\sqrt{x^2+A})(\sqrt{x^2+A})} = \boxed{\dfrac{1}{\sqrt{x^2+A}}}$

注)　$(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}$

(9) $y' = (x)'\sqrt{x^2+A} + x(\sqrt{x^2+A})' + A(\log(x+\sqrt{x^2+A}))'$

$= \sqrt{x^2+A} + x\dfrac{2x}{2\sqrt{x^2+A}} + \dfrac{A}{\sqrt{x^2+A}} = \dfrac{(\sqrt{x^2+A})^2 + x^2 + A}{\sqrt{x^2+A}}$

$= \dfrac{2(x^2+A)}{\sqrt{x^2+A}} = \boxed{2\sqrt{x^2+A}}$

注)　$(\sqrt{a})^2 = a, \qquad \dfrac{a}{\sqrt{a}} = \sqrt{a}$

レポート 4 解答　　8 点

(1) $y' = (\cos(2x-3))(2x-3)' = \boxed{2\cos(2x-3)}$

(2) $y' = (-\sin x^3)(x^3)' = \boxed{-3x^2\sin(x^3)}$

(3) $y' = 4\sin^3 x(\sin x)' = \boxed{4\sin^3 x\cos x}$
注　$\sin^n x = (\sin x)^n$

(4) $y' = \dfrac{(\cos x)'\sin x - \cos x(\sin x)'}{\sin^2 x} = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \boxed{-\dfrac{1}{\sin^2 x}}$
注　$\dfrac{f}{g} = \dfrac{f'g - fg'}{g^2}, \qquad \sin^2 x + \cos^2 x = 1$

(5) $y' = \dfrac{(x^2-x+1)'}{\cos^2(x^2-x+1)} = \boxed{\dfrac{2x-1}{\cos^2(x^2-x+1)}}$

(6) $y' = (\sin^2 x)'\cos 3x + \sin^2 x(\cos 3x)'$
$= 2\sin x(\sin x)'\cos 3x + \sin^2 x(-\sin 3x)(3x)' = \boxed{2\sin x\cos x\cos 3x - 3\sin^2 x\sin 3x}$

(7) $y' = \dfrac{(\cos x)'}{\cos x} = -\dfrac{\sin x}{\cos x} = \boxed{-\tan x}$

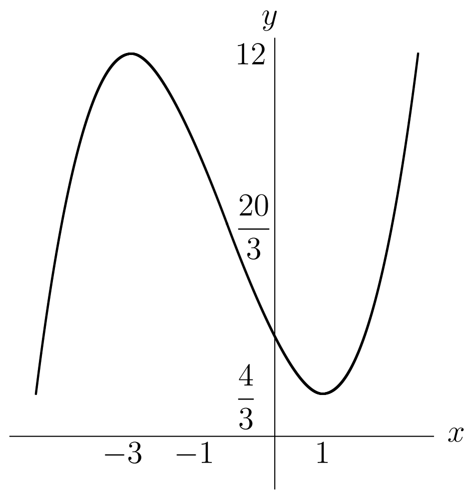
(8) $y' = (e^{2x})'\sin 3x + e^{2x}(\sin 3x)' = e^{2x}(2x)'\sin 3x + e^{2x}\cos 3x(3x)' = \boxed{2e^{2x}\sin 3x + 3e^{2x}\cos 3x}$

レポート 5 解答 No. 1

(1) $y = \dfrac{1}{3}x^3 + x^2 - 3x + 3$
$y' = x^2 + 2x - 3 = (x+3)(x-1)$
$y'' = 2x + 2 = 2(x+1)$
$y' = 0$ となるのは $x = -3, 1$
$y'' = 0$ となるのは $x = -1$

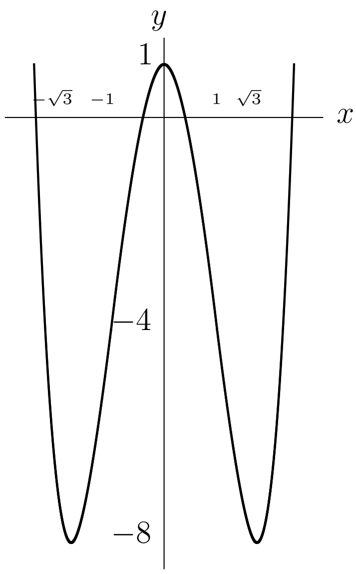
x	$\cdots$	-3	$\cdots$	-1	$\cdots$	1	$\cdots$
y'	$+$	0	$-$	$-$	$-$	0	$+$
y''	$-$	$-$	$-$	0	$+$	$+$	$+$
y	$\nearrow$	12 極大	$\searrow$	$\dfrac{20}{3}$ 変曲点	$\searrow$	$\dfrac{4}{3}$ 極小	$\nearrow$

右のグラフで、y 軸は x 軸の $\dfrac{2}{3}$ 倍



(2) $y = x^4 - 6x^2 + 1$
$y' = 4x^3 - 12x = 4x(x^2 - 3)$
$y'' = 12x^2 - 12 = 12(x^2 - 1)$
$y' = 0$ となるのは $x = 0, \pm\sqrt{3}$
$y'' = 0$ となるのは $x = \pm 1$

x	$\cdots$	$-\sqrt{3}$	$\cdots$	-1	$\cdots$	0	$\cdots$	1	$\cdots$	$\sqrt{3}$	$\cdots$
y'	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$
y''	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
y	$\searrow$	-8 極小	$\nearrow$	-4 変曲点	$\nearrow$	1 極大	$\searrow$	-4 変曲点	$\searrow$	-8 極小	$\nearrow$



レポート 5 解答 No. 2

(3) $y = \dfrac{1}{x} + x$

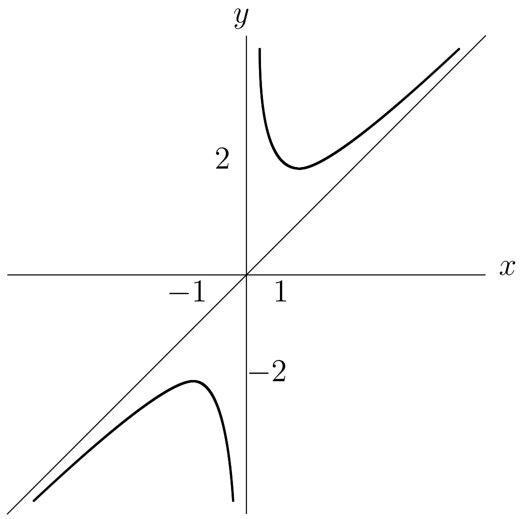
$y' = -\dfrac{1}{x^2} + 1$

$y'' = \dfrac{2}{x^3}$

$y' = 0$ となるのは $x = \pm 1$

$y'' = 0$ となる x はない

x	$\cdots$	-1	$\cdots$	0	$\cdots$	1	$\cdots$
y'	$+$	0	$-$	$\times$	$-$	0	$+$
y''	$-$	$-$	$-$	$\times$	$+$	$+$	$+$
y	$\curvearrowright$	-2 極大	↘	$\times$	↘	2 極小	↗



(4) $y = xe^{-2x}$

$y' = (1-2x)e^{-2x}$

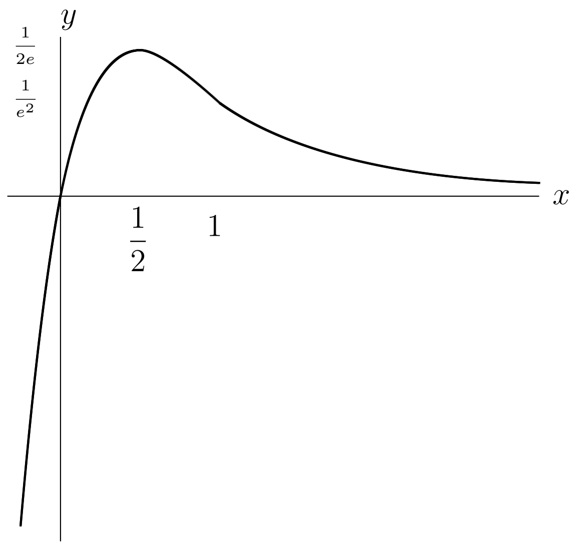
$y'' = 4(x-1)e^{-2x}$

$y' = 0$ となるのは $x = \dfrac{1}{2}$

$y'' = 0$ となるのは $x = 1$

x	$\cdots$	$\frac{1}{2}$	$\cdots$	1	$\cdots$
y'	$+$	0	$-$	$-$	$-$
y''	$-$	$-$	$-$	0	$+$
y	$\curvearrowright$	$\frac{1}{2e}$ 極大	↘	$\frac{1}{e^2}$ 変曲点	↘

右のグラフで、y 軸は x 軸の 5 倍

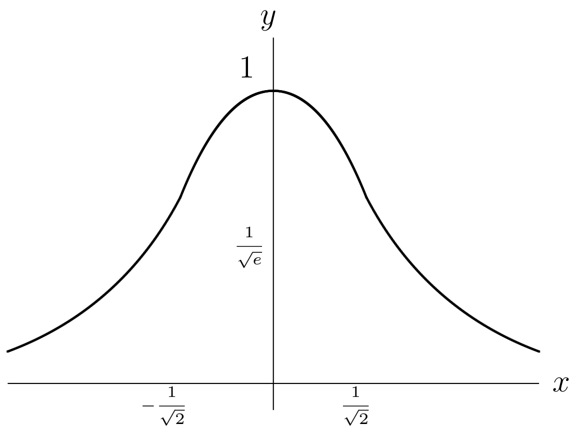


レポート 5 解答 No. 3

(5) $y = e^{-x^2}$
$y' = -2xe^{-x^2}$
$y'' = 2(2x^2-1)e^{-x^2}$
$y' = 0$ となるのは $x = 0$
$y'' = 0$ となるのは $x = \pm\dfrac{1}{\sqrt{2}}$

x	$\cdots$	$-\dfrac{1}{\sqrt{2}}$	$\cdots$	0	$\cdots$	$\dfrac{1}{\sqrt{2}}$	$\cdots$
y'	$+$	$+$	$+$	0	$-$	$-$	$-$
y''	$+$	0	$-$	$-$	$-$	0	$+$
y	↗	$\dfrac{1}{\sqrt{e}}$ 変曲点	↗	1 極大	↘	$\dfrac{1}{\sqrt{e}}$ 変曲点	↘

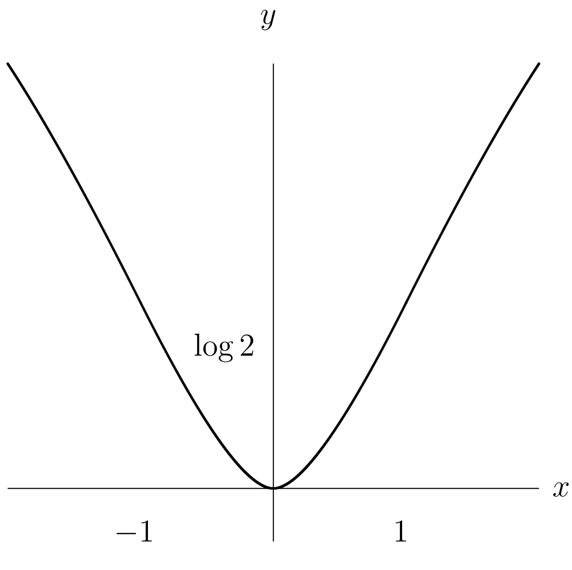
右のグラフで、y 軸は x 軸の 2 倍



(6) $y = \log(1+x^2)$
$y' = \dfrac{2x}{1+x^2}$
$y'' = \dfrac{2(1-x^2)}{1+x^2}$
$y' = 0$ となるのは $x = 0$
$y'' = 0$ となるのは $x = \pm 1$

x	$\cdots$	-1	$\cdots$	0	$\cdots$	1	$\cdots$
y'	$-$	$-$	$-$	0	$+$	$+$	$+$
y''	$-$	0	$+$	$+$	$+$	0	$-$
y	↘	$\log 2$ 変曲点	↘	0 極小	↗	$\log 2$ 変曲点	↗

右のグラフで、y 軸は x 軸の 2 倍

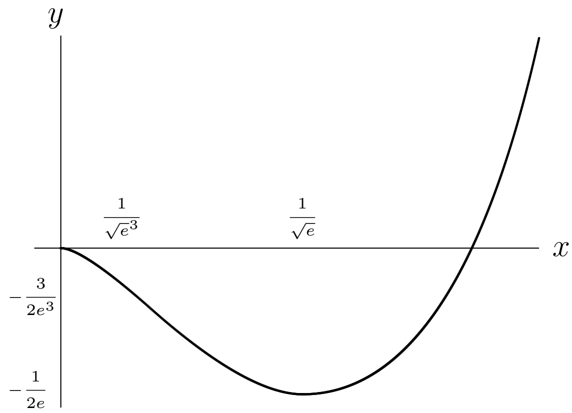


レポート 5 解答 No. 4

(7) $y = x^2 \log x$
$y' = x(2\log x + 1)$
$y'' = 2\log x + 3$
$y' = 0$ となるのは $x = 0,\ \dfrac{1}{\sqrt{e}}$
$y'' = 0$ となるのは $x = \dfrac{1}{\sqrt{e^3}}$

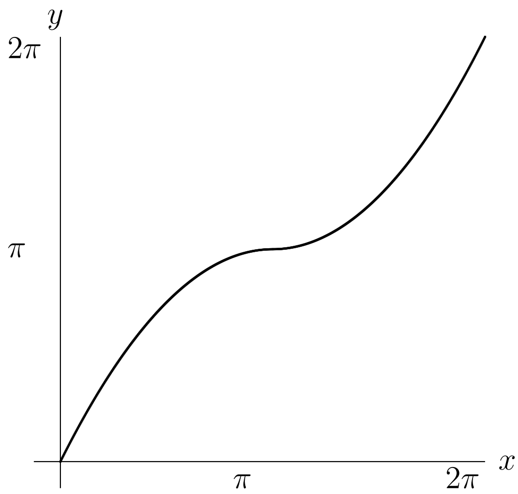
x	0	$\cdots$	$\dfrac{1}{\sqrt{e^3}}$	$\cdots$	$\dfrac{1}{\sqrt{e}}$	$\cdots$
y'	0	$-$	$-$	$-$	0	$+$
y''	$\times$	$-$	0	$+$	$+$	$+$
y	0	↘	$-\dfrac{3}{2e^3}$ 変曲点	↘	$-\dfrac{1}{2e}$ 極小	↗

右のグラフで、y 軸は x 軸の 2 倍



(8) $y = x + \sin x \quad (0 \leq x \leq 2\pi)$
$y' = 1 + \cos x \geq 0$
$y'' = -\sin x$
$y' = 0$ となるのは $x = \pi$
$y'' = 0$ となるのは $x = 0,\ \pi,\ 2\pi$

x	0	$\cdots$	π	$\cdots$	2π
y'	$+$	$+$	0	$+$	$+$
y''	0	$-$	0	$+$	0
y	0	↗	π 変曲点	↗	2π



レポート 6　解答

(1) $\displaystyle\int \frac{1}{x^2+2x}\,dx \int \frac{1}{(x+1)^2-1}\,dx = \boxed{\frac{1}{2}\log\left|\frac{x}{x+2}\right| + C}$

(2) $\displaystyle\int \frac{1}{2x^2+4x+4}\,dx = \frac{1}{2}\int \frac{1}{(x+1)^2+1}\,dx = \boxed{\frac{1}{2}\,\mathrm{Tan}^{-1}(x+1) + C}$

(3) $\displaystyle\int \frac{1}{\sqrt{-4x^2+8x-3}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2-(x-1)^2}} = \boxed{\frac{1}{2}\,\mathrm{Sin}^{-1}\,2(x-1) + C}$

(4) $\displaystyle\int \frac{1}{\sqrt{4x^2-8x+3}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{(x-1)^2-\frac{1}{4}}}\,dx = \frac{1}{2}\log\left|(x-1)+\sqrt{(x-1)^2-\frac{1}{4}}\right| + C$

$\displaystyle = \boxed{\frac{1}{2}\log\left|2x-2+\sqrt{4x^2-8x+3}\right| + C}$

注 $\displaystyle\frac{1}{2}\log\frac{|f(x)|}{2} + C = \frac{1}{2}\log|f(x)| - \frac{1}{2}\log 2 + C$ であり、

$\displaystyle-\frac{1}{2}\log 2 + C$ を新たに C としている。

(5) $\displaystyle\int \sqrt{-x^2+4x-1}\,dx = \int \sqrt{\sqrt{3}^2-(x-2)^2}\,dx$

$\displaystyle = \boxed{\frac{1}{2}\left\{(x-2)\sqrt{-x^2+4x-1} + 3\,\mathrm{Sin}^{-1}\frac{x-2}{\sqrt{3}}\right\} + C}$

(6) $\displaystyle\int \sqrt{x^2-4x+1}\,dx = \int \sqrt{(x-2)^2-3}\,dx$

$\displaystyle = \boxed{\frac{1}{2}\left\{(x-2)\sqrt{x^2-4x+1} - 3\log\left|x-2+\sqrt{x^2-4x+1}\right|\right\} + C}$

レポート 7 解答

問 1. (1) $= \displaystyle\int t^3(x-1)\frac{1}{2x-2}\,dt = \frac{1}{2}\int t^3\,dt = \frac{1}{2}\frac{t^4}{4} + C = \boxed{\frac{(x^2-2x-1)^4}{8} + C}$

注 $\displaystyle\frac{dt}{dx} = (x^2-2x-1)' = 2x-2, \qquad dx = \frac{1}{(2x-2)}\,dt$

(2) $= \displaystyle\int \frac{\cos x}{t}\frac{1}{\cos x}\,dt = \int \frac{1}{t}\,dt = \log|t| + C = \boxed{\log|\sin x| + C}$

(3) $= \displaystyle\int (t^2-1)t2t\,dt = \int 2t^4 - 2t^2\,dt = \frac{2}{5}t^5 - \frac{2}{3}t^3 + C = \boxed{\frac{2}{5}\sqrt{x+1}^5 - \frac{2}{3}\sqrt{x+1}^3 + C}$

注 $\displaystyle\frac{dx}{dt} = (t^2-1)' = 2t, \qquad dx = 2t\,dt$

(4) $= \displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{\sin^2 x}\,dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{1-\cos^2 x}\,dx = \int_{\frac{1}{2}}^{0} \frac{\sin x}{1-t^2}\frac{1}{(-\sin x)}\,dt = \int_{\frac{1}{2}}^{0} \frac{1}{t^2-1}\,dt$

$= \displaystyle\left[\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|\right]_{\frac{1}{2}}^{0} = \frac{1}{2}\left(\log 1 - \log\frac{\frac{1}{2}}{\frac{3}{2}}\right) = \boxed{\frac{1}{2}\log 3}$

注 $\displaystyle\cos^2 x + \sin^2 x = 1, \qquad \cos\frac{\pi}{2} = 0, \qquad \cos\frac{\pi}{3} = \frac{1}{2}, \qquad \log 1 = 0, \qquad \log\frac{1}{a} = -\log a$

(5) $= \displaystyle\int_0^1 \frac{t^2}{x}x\,dt = \int_0^1 t^2\,dt = \left[\frac{t^3}{3}\right]_0^1 = \boxed{\frac{1}{3}}$

注 $\displaystyle\frac{dt}{dx} = (\log x)' = \frac{1}{x}, \qquad dx = x\,dt, \qquad \log e = 1, \qquad \log 1 = 0$

(6) $= \displaystyle\int_1^{\sqrt{3}} \frac{e^x}{t^2+1}\frac{1}{e^x}\,dt = \int_1^{\sqrt{3}} \frac{1}{1+t^2}\,dt = \left[\mathrm{Tan}^{-1}\,t\right]_1^{\sqrt{3}} = \mathrm{Tan}^{-1}\sqrt{3} - \mathrm{Tan}^{-1}1 = \frac{\pi}{3} - \frac{\pi}{4} = \boxed{\frac{\pi}{12}}$

注 $e^0 = 1, \qquad e^{\log a} = a$

問 2. (1) $\displaystyle\int \frac{1}{\sin x}\,dx = \int \frac{1}{\left(\frac{2t}{1+t^2}\right)}\frac{2}{(1+t^2)}\,dt = \int \frac{1}{t}\,dt = \log|t| + C = \boxed{\log\left|\tan\frac{x}{2}\right| + C}$

(2) $\displaystyle\int \frac{1}{2\cos x + 3}\,dx = \int \frac{1}{\left(2\frac{1-t^2}{1+t^2} + 3\right)}\frac{2}{(1+t^2)}\,dt = \int \frac{2}{t^2+5}\,dt = \frac{2}{\sqrt{5}}\mathrm{Tan}^{-1}\frac{t}{\sqrt{5}} + C$

$\displaystyle= \boxed{\frac{2}{\sqrt{5}}\mathrm{Tan}^{-1}\left(\frac{1}{\sqrt{5}}\tan\frac{x}{2}\right) + C}$

(3) $\displaystyle\int \frac{1}{2\sin x + 3}\,dx = \int \frac{1}{\left(2\frac{2t}{1+t^2} + 3\right)}\frac{2}{(1+t^2)}\,dt = \int \frac{2}{3t^2+4t+3}\,dt = \frac{2}{3}\int \frac{1}{\left(t+\frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2}\,dt$

$\displaystyle= \frac{2}{3}\frac{1}{\frac{\sqrt{5}}{3}}\mathrm{Tan}^{-1}\frac{t+\frac{2}{3}}{\frac{\sqrt{5}}{3}} + C = \boxed{\frac{2}{\sqrt{5}}\mathrm{Tan}^{-1}\left(\frac{3\tan\frac{x}{2}+2}{\sqrt{5}}\right) + C}$

レポート 8 解答 No.1 10 点

問 1. (1) $= \displaystyle\int \left(\frac{x^3}{3}\right)' \log x\, dx = \frac{x^3}{3}\log x - \int \frac{x^3}{3}(\log x)'\, dx$

$\displaystyle = \frac{x^3\log x}{3} - \int \frac{x^3}{3}\frac{1}{x}\, dx = \frac{x^3\log x}{3} - \int \frac{x^2}{3}\, dx = \boxed{\frac{x^3\log x}{3} - \frac{x^3}{9} + C},$ 　　注 $\displaystyle\int x^2 dx = \frac{x^3}{3} + C$

(2) $= \displaystyle\int x^2(\sin x)'\, dx = x^2\sin x - \int (x^2)'\sin x\, dx = x^2\sin x - 2\int x\sin x\, dx$

$\displaystyle = x^2\sin x - 2\int x(-\cos x)'\, dx = x^2\sin x - 2\left(-x\cos x + \int (x)'\cos x\, dx\right)$

$\displaystyle = x^2\sin x + 2x\cos x - 2\int \cos x\, dx = \boxed{x^2\sin x + 2x\cos x - 2\sin x + C},$

注 $\displaystyle\int \cos x dx = \sin x + C,$ 　　$\displaystyle\int \sin x\, dx = -\cos x + C$

(3) $= \displaystyle\int x\left(\frac{e^{3x}}{3}\right)' dx = x\frac{e^{3x}}{3} - \int (x)'\frac{e^{3x}}{3}\, dx = x\frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}\, dx = \boxed{x\frac{e^{3x}}{3} - \frac{e^{3x}}{9} + C}$

注 $\displaystyle\int e^{3x} dx = \frac{e^{3x}}{3} + C$

(4) $= \displaystyle\int_1^e \left(\frac{x^2}{2}\right)' \log x\, dx = \left[\frac{x^2}{2}\log x\right]_1^e - \int_1^e \frac{x^2}{2}(\log x)'\, dx$

$\displaystyle = \frac{e^2}{2} - \int_1^e \frac{x^2}{2}\frac{1}{x}\, dx = \frac{e^2}{2} - \int_1^e \frac{x}{2}\, dx = \frac{e^2}{2} - \left[\frac{x^2}{4}\right]_1^e = \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} = \boxed{\frac{e^2+1}{4}}$

注 $\displaystyle\int x dx = \frac{x^2}{2} + C,$ 　　$\log e = 1,$ 　　$\log 1 = 0,$ 　　$\displaystyle\frac{a}{2} - \frac{a}{4} = \frac{2a-a}{4} = \frac{a}{4}$

(5) $= \displaystyle\int_0^{\frac{\pi}{6}} x\left(-\frac{\cos 2x}{2}\right)' dx = \left[-x\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{6}} + \frac{1}{2}\int_0^{\frac{\pi}{6}} (x)'\cos 2x\, dx = -\frac{1}{2}\frac{\pi}{6}\cos\frac{\pi}{3} + \frac{1}{2}\int_0^{\frac{\pi}{6}} \cos 2x\, dx$

$\displaystyle = -\frac{\pi}{24} + \frac{1}{2}\left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{6}} = -\frac{\pi}{24} + \frac{1}{4}\sin\frac{\pi}{3} = \boxed{-\frac{\pi}{24} + \frac{\sqrt{3}}{8}}$

注 $\displaystyle\int \sin 2x dx = -\frac{\cos 2x}{2} + C,$ 　　$\cos 0 = 1,$ 　　$\displaystyle\cos\frac{\pi}{3} = \frac{1}{2},$ 　　$\sin 0 = 0,$ 　　$\displaystyle\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

(6) $= \displaystyle\int_0^1 x^3(e^x)'\, dx = \left[x^3e^x\right]_0^1 - \int_0^1 (x^3)'e^x\, dx = e - 3\int_0^1 x^2(e^x)'\, dx$

$\displaystyle = e - 3\left(\left[x^2e^x\right]_0^1 - \int_0^1 (x^2)'e^x\, dx\right) = e - 3\left(e - \int_0^1 2xe^x\, dx\right) = -2e + 6\int_0^1 x(e^x)'\, dx$

$\displaystyle = -2e + 6\left(\left[xe^x\right]_0^1 - \int_0^1 (x)'e^x\, dx\right) = -2e + 6\left(e - \left[e^x\right]_0^1\right) = -2e + 6e - 6(e-1) = \boxed{-2e+6}$

注 $\displaystyle\int e^x dx = e^x + C,$ 　　$e^0 = 1,$ 　　$e^1 = e$

レポート 8 解答 No.2

問 2. (1) 部分分数分解して、$\dfrac{x}{(x-2)(x+1)} = \dfrac{A}{x-2} + \dfrac{B}{x+1}$

$$A = \left.\frac{x}{(x-2)(x+1)}(x-2)\right|_{x=2} = \frac{2}{3},$$

$$B = \left.\frac{x}{(x-2)(x+1)}(x+1)\right|_{x=-1} = \frac{1}{3}$$

$$\begin{aligned}\int \frac{x}{(x-2)(x+1)}\,dx &= \int \left\{\frac{\frac{2}{3}}{x-2} + \frac{\frac{1}{3}}{x+1}\right\} dx \\ &= \frac{2}{3}\log|x-2| + \frac{1}{3}\log|x+1| + C \\ &= \boxed{\log\left|\sqrt[3]{(x-2)^2(x+1)}\right| + C}\end{aligned}$$

(2) 部分分数分解して、$\dfrac{x+3}{(x-1)(x+1)(x+2)} = \dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{C}{x+2}$

$$\begin{aligned}A &= \left.\frac{x+3}{(x-1)(x+1)(x+2)}(x-1)\right|_{x=1} = \frac{2}{3}, \\ B &= \left.\frac{x+3}{(x-1)(x+1)(x+2)}(x+1)\right|_{x=-1} = -1, \\ C &= \left.\frac{x+3}{(x-1)(x+1)(x+2)}(x+2)\right|_{x=-2} = \frac{1}{3},\end{aligned}$$

$$\begin{aligned}\int \frac{x+3}{(x-1)(x+1)(x+2)}\,dx &= \int \left\{\frac{\frac{2}{3}}{x-1} - \frac{1}{x+1} + \frac{\frac{1}{3}}{x+2}\right\} dx \\ &= \frac{2}{3}\log|x-1| - \log|x+1| + \frac{1}{3}\log|x+2| + C \\ &= \boxed{\log\left|\frac{\sqrt[3]{(x-1)^2(x+2)}}{x+1}\right| + C}\end{aligned}$$

レポート 8　解答 No.3

(3) 部分分数分解して、$\dfrac{x^2+x+1}{(x-1)x^2} = \dfrac{A}{x-1} + \dfrac{B_2}{x^2} + \dfrac{B_1}{x}$

$$\begin{aligned}
A &= \left.\frac{x^2+x+1}{(x-1)x^2}(x-1)\right|_{x=1} = 3, \\
B_2 &= \left.\frac{x^2+x+1}{(x-1)x^2}x^2\right|_{x=0} = -1, \\
B_1 &= \left.\frac{1}{1!}\frac{d}{dx}\left\{\frac{x^2+x+1}{(x-1)x^2}x^2\right\}\right|_{x=0} = \left.\frac{(x^2+x+1)'(x-1)-(x^2+x+1)(x-1)'}{(x-1)^2}\right|_{x=0} \\
&= \left.\frac{x^2-2x-2}{(x-1)^2}\right|_{x=0} = -2,
\end{aligned}$$

$$\begin{aligned}
\int \frac{x^2+x+1}{(x-1)x^2}\,dx &= \int \left\{\frac{3}{x-1} - \frac{1}{x^2} - \frac{2}{x}\right\} dx \\
&= 3\log|x-1| + \frac{1}{x} - 2\log|x| + C \\
&= \boxed{\frac{1}{x} + \log\left|\frac{(x-1)^3}{x^2}\right| + C}
\end{aligned}$$

(4) 部分分数分解して、$\dfrac{x+1}{x(x+2)^3} = \dfrac{A}{x} + \dfrac{B_3}{(x+2)^3} + \dfrac{B_2}{(x+2)^2} + \dfrac{B_1}{(x+2)}$

$$\begin{aligned}
A &= \left.\frac{x+1}{x(x+2)^3}x\right|_{x=0} = \frac{1}{8}, \\
B_3 &= \left.\frac{x+1}{x(x+2)^3}(x+2)^3\right|_{x=-2} = \frac{1}{2}, \\
B_2 &= \left.\frac{1}{1!}\frac{d}{dx}\left\{\frac{x+1}{x(x+2)^3}(x+2)^3\right\}\right|_{x=-2} = \left.\frac{(x+1)'x-(x+1)(x)'}{x^2}\right|_{x=-2} \\
&= \left.\frac{-1}{x^2}\right|_{x=-2} = -\frac{1}{4}, \\
B_1 &= \left.\frac{1}{2!}\frac{d^2}{dx^2}\left\{\frac{x+1}{x(x+2)^3}(x+2)^3\right\}\right|_{x=-2} = \left.\frac{1}{2}\frac{d}{dx}\left\{\frac{-1}{x^2}\right\}\right|_{x=-2} = \left.\frac{1}{2}\frac{2}{x^3}\right|_{x=-2} = -\frac{1}{8},
\end{aligned}$$

$$\begin{aligned}
\int \frac{x+1}{x(x+2)^3}\,dx &= \int \left\{\frac{\frac{1}{8}}{x} + \frac{\frac{1}{2}}{(x+2)^3} + \frac{-\frac{1}{4}}{(x+2)^2} + \frac{-\frac{1}{8}}{(x+2)}\right\} dx \\
&= \frac{1}{8}\log|x| - \frac{1}{4}\frac{1}{(x+2)^2} + \frac{1}{4}\frac{1}{(x+2)} - \frac{1}{8}\log|x+2| + C \\
&= \boxed{\frac{(x+1)}{4(x+2)^2} + \log\sqrt[8]{\left|\frac{x}{x+2}\right|} + C}
\end{aligned}$$

レポート 9 解答 No.1 18 点

(1) $z_x = \boxed{6x^2 - 8xy^3 + 3y^4}$,　$z_y = \boxed{-12x^2y^2 + 12xy^3 - 10y^4}$

$z_{xx} = \boxed{12x - 8y^3}$, $z_{xy} = \boxed{-24xy^2 + 12y^3}$, $z_{yx} = \boxed{-24xy^2 + 12y^3}$, $z_{yy} = \boxed{-24x^2y + 36xy^2 - 40y^3}$

(2) $z_x = \dfrac{\partial_x(x^2+xy)}{2\sqrt{x^2+xy}} = \boxed{\dfrac{2x+y}{2\sqrt{x^2+xy}}}$,　$z_y = \dfrac{\partial_y(x^2+xy)}{2\sqrt{x^2+xy}} = \boxed{\dfrac{x}{2\sqrt{x^2+xy}}}$

注　以下は上の z_x, z_y の計算結果を使用する。

$$z_{xx} = \frac{\partial_x(2x+y)2\sqrt{x^2+xy} - (2x+y)\partial_x(2\sqrt{x^2+xy})}{4(\sqrt{x^2+xy})^2} = \frac{4\sqrt{x^2+xy} - (2x+y)2\frac{(2x+y)}{2\sqrt{x^2+xy}}}{4(x^2+xy)}$$

$$= \frac{4(\sqrt{x^2+xy})^2 - (2x+y)^2}{4(x^2+xy)\sqrt{x^2+xy}} = \boxed{-\frac{y^2}{4(x^2+xy)\sqrt{x^2+xy}}}$$

$$z_{xy} = \frac{\partial_y(2x+y)2\sqrt{x^2+xy} - (2x+y)\partial_y(2\sqrt{x^2+xy})}{4(\sqrt{x^2+xy})^2} = \frac{2\sqrt{x^2+xy} - (2x+y)2\frac{x}{2\sqrt{x^2+xy}}}{4(x^2+xy)}$$

$$= \frac{2(\sqrt{x^2+xy})^2 - (2x+y)x}{4(x^2+xy)\sqrt{x^2+xy}} = \boxed{\frac{xy}{4(x^2+xy)\sqrt{x^2+xy}}}$$

$$z_{yx} = \frac{\partial_x(x)2\sqrt{x^2+xy} - x\partial_x(2\sqrt{x^2+xy})}{4(\sqrt{x^2+xy})^2} = \frac{2\sqrt{x^2+xy} - x2\frac{(2x+y)}{2\sqrt{x^2+xy}}}{4(x^2+xy)}$$

$$= \frac{2(\sqrt{x^2+xy})^2 - x(2x+y)}{4(x^2+xy)\sqrt{x^2+xy}} = \boxed{\frac{xy}{4(x^2+xy)\sqrt{x^2+xy}}}$$

$$z_{yy} = \frac{\partial_y(x)2\sqrt{x^2+xy} - x\partial_y(2\sqrt{x^2+xy})}{4(\sqrt{x^2+xy})^2} = \frac{0 - x2\frac{x}{2\sqrt{x^2+xy}}}{4(x^2+xy)}$$

$$= \boxed{-\frac{x^2}{4(x^2+xy)\sqrt{x^2+xy}}}$$

(3) $z_x = \dfrac{\partial_x(x^2+y^2)}{x^2+y^2} = \boxed{\dfrac{2x}{x^2+y^2}}$,　$z_y = \dfrac{\partial_y(x^2+y^2)}{x^2+y^2} = \boxed{\dfrac{2y}{x^2+y^2}}$

$$z_{xx} = \frac{\partial_x(2x)(x^2+y^2) - 2x\partial_x(x^2+y^2)}{(x^2+y^2)^2} = \frac{2(x^2+y^2) - 2x(2x)}{(x^2+y^2)^2} = \boxed{\frac{2y^2-2x^2}{(x^2+y^2)^2}}$$

$$z_{xy} = \frac{\partial_y(2x)(x^2+y^2) - 2x\partial_y(x^2+y^2)}{(x^2+y^2)^2} = \frac{0 - 2x(2y)}{(x^2+y^2)^2} = \boxed{-\frac{4xy}{(x^2+y^2)^2}}$$

$$z_{yx} = \frac{\partial_x(2y)(x^2+y^2) - 2y\partial_x(x^2+y^2)}{(x^2+y^2)^2} = \frac{0 - 2y(2x)}{(x^2+y^2)^2} = \boxed{-\frac{4xy}{(x^2+y^2)^2}}$$

$$z_{yy} = \frac{\partial_y(2y)(x^2+y^2) - 2y\partial_y(x^2+y^2)}{(x^2+y^2)^2} = \frac{2(x^2+y^2) - 2y(2y)}{(x^2+y^2)^2} = \boxed{\frac{2x^2-2y^2}{(x^2+y^2)^2}}$$

レポート 9 解答 No.2 18 点

(4) 注　z_x は公式 $(x^n)' = nx^{n-1}$ を、z_y は公式 $(a^x)' = a^x \log a$ を使用

$z_x = \boxed{yx^{y-1}}, \qquad z_y = \boxed{x^y \log x}$

$z_{xx} = \boxed{y(y-1)x^{y-2}}$

$z_{xy} = \partial_y(y)x^{y-1} + y\partial_y(x^{y-1}) = x^{y-1} + yx^{y-1}\log x \partial_y(y-1) = \boxed{x^{y-1} + yx^{y-1}\log x}$

$z_{yx} = \partial_x(x^y)\log x + x^y\partial_x(\log x) = yx^{y-1}\log x + x^y\dfrac{1}{x} = \boxed{yx^{y-1}\log x + x^{y-1}}$

$z_{yy} = \partial_y(x^y)\log x = \boxed{x^y(\log x)^2}$

(5) 注　$\partial_x\left(\dfrac{x}{y}\right) = \dfrac{1}{y}, \qquad \partial_y\left(\dfrac{x}{y}\right) = \partial_y(xy^{-1}) = x(-1)y^{-2} = -\dfrac{x}{y^2}$

$$z_x = \frac{1}{1+\left(\frac{x}{y}\right)^2}\partial_x\left(\frac{x}{y}\right) = \frac{y^2}{x^2+y^2}\left(\frac{1}{y}\right) = \boxed{\frac{y}{x^2+y^2}}$$

$$z_y = \frac{1}{1+\left(\frac{x}{y}\right)^2}\partial_y\left(\frac{x}{y}\right) = \frac{y^2}{x^2+y^2}\left(-\frac{x}{y^2}\right) = \boxed{-\frac{x}{x^2+y^2}}$$

$$z_{xx} = \frac{\partial_x(y)(x^2+y^2) - y\partial_x(x^2+y^2)}{(x^2+y^2)^2} = \frac{0-y(2x)}{(x^2+y^2)^2} = \boxed{-\frac{2xy}{(x^2+y^2)^2}}$$

$$z_{xy} = \frac{\partial_y(y)(x^2+y^2) - y\partial_y(x^2+y^2)}{(x^2+y^2)^2} = \frac{(x^2+y^2)-y(2y)}{(x^2+y^2)^2} = \boxed{\frac{x^2-y^2}{(x^2+y^2)^2}}$$

$$z_{yx} = -\frac{\partial_x(x)(x^2+y^2) - x\partial_x(x^2+y^2)}{(x^2+y^2)^2} = -\frac{(x^2+y^2)-x(2x)}{(x^2+y^2)^2} = \boxed{\frac{x^2-y^2}{(x^2+y^2)^2}}$$

$$z_{yy} = -\frac{\partial_y(x)(x^2+y^2) - x\partial_y(x^2+y^2)}{(x^2+y^2)^2} = -\frac{0-x(2y)}{(x^2+y^2)^2} = \boxed{\frac{2xy}{(x^2+y^2)^2}}$$

(6) $z_x = e^{2x+3y}\partial_x(2x+3y) = \boxed{2e^{2x+3y}}, \qquad z_y = e^{2x+3y}\partial_y(2x+3y) = \boxed{3e^{2x+3y}}$

$z_{xx} = 2e^{2x+3y}\partial_x(2x+3y) = \boxed{4e^{2x+3y}}$

$z_{xy} = 2e^{2x+3y}\partial_y(2x+3y) = \boxed{6e^{2x+3y}}$

$z_{yx} = 3e^{2x+3y}\partial_x(2x+3y) = \boxed{6e^{2x+3y}}$

$z_{yy} = 3e^{2x+3y}\partial_y(2x+3y) = \boxed{9e^{2x+3y}}$

レポート 10 解答 1 20 点

問 1 (1)

$$
\begin{aligned}
f_x &= \boxed{2x}, & f_y &= \boxed{2y} \\
\frac{\partial x}{\partial u} &= \boxed{2e^{2u}\cos 3v}, & \frac{\partial y}{\partial u} &= \boxed{2e^{2u}\sin 3v} \\
\frac{\partial x}{\partial v} &= \boxed{-3e^{2u}\sin 3v}, & \frac{\partial y}{\partial v} &= \boxed{3e^{2u}\cos 3v}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f}{\partial u} &= f_x\frac{\partial x}{\partial u} + f_y\frac{\partial y}{\partial u} = 2x2e^{2u}\cos 3v + 2y2e^{2u}\sin 3v = 4e^{4u}\cos^2 3v + 4e^{4u}\sin^2 3v = \boxed{4e^{4u}} \\
\frac{\partial f}{\partial v} &= f_x\frac{\partial x}{\partial v} + f_y\frac{\partial y}{\partial v} = 2x(-3e^{2u}\sin 3v) + 2y3e^{2u}\cos 3v = -6e^{4u}\cos 3v\sin 3v + 6e^{4u}\sin 3v\cos 3v = \boxed{0}
\end{aligned}
$$

注　$(e^{ax})' = ae^{ax}, (\cos ax)' = -a\cos ax, (\sin ax)' = a\cos ax, \cos^2 x + \sin^2 x = 1$

$f(u,v) = x^2 + y^2 = (e^{2u}\cos v)^2 + (e^{2u}\sin v)^2 = e^{4u},\quad \dfrac{\partial f}{\partial u} = 4e^{4u},\quad \dfrac{\partial f}{\partial v} = 0$

(2)

$$
\begin{aligned}
f_x &= \boxed{-\frac{y}{x^2}}, & f_y &= \boxed{\frac{1}{x}} \\
\frac{\partial x}{\partial s} &= \boxed{\frac{\sqrt{t}}{3\sqrt[3]{s}^2}}, & \frac{\partial y}{\partial s} &= \boxed{\log t} \\
\frac{\partial x}{\partial t} &= \boxed{\frac{\sqrt[3]{s}}{2\sqrt{t}}}, & \frac{\partial y}{\partial t} &= \boxed{\frac{s}{t}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f}{\partial s} &= f_x\frac{\partial x}{\partial s} + f_y\frac{\partial y}{\partial s} = -\frac{y}{x^2}\frac{\sqrt{t}}{3\sqrt[3]{s}^2} + \frac{1}{x}\log t \\
&= -\frac{s\log t}{\sqrt[3]{s}^2 t}\frac{\sqrt{t}}{3\sqrt[3]{s}^2} + \frac{1}{\sqrt[3]{s}\sqrt{t}}\log t = -\frac{\log t}{3\sqrt[3]{s}\sqrt{t}} + \frac{\log t}{\sqrt[3]{s}\sqrt{t}} = \boxed{\frac{2\log t}{3\sqrt[3]{s}\sqrt{t}}} \\
\frac{\partial f}{\partial t} &= f_x\frac{\partial x}{\partial t} + f_y\frac{\partial y}{\partial t} = -\frac{y}{x^2}\frac{\sqrt[3]{s}}{2\sqrt{t}} + \frac{1}{x}\frac{s}{t} \\
&= -\frac{s\log t}{\sqrt[3]{s}^2 t}\frac{\sqrt[3]{s}}{2\sqrt{t}} + \frac{1}{\sqrt[3]{s}\sqrt{t}}\frac{s}{t} = \boxed{-\frac{\sqrt[3]{s}^2}{2\sqrt{t}^3}\log t + \frac{\sqrt[3]{s}^2}{\sqrt{t}^3}} \\
&= \frac{\sqrt[3]{s}^2}{2\sqrt{t}^3}(2 - \log t)
\end{aligned}
$$

注　$\left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2},\ (\sqrt[3]{x})' = (x^{\frac{1}{3}})' = \dfrac{1}{3}x^{-\frac{2}{3}} = \dfrac{1}{3\sqrt[3]{x}^2},\ (\sqrt{x})' = \dfrac{1}{2\sqrt{x}},\ (\log x)' = \dfrac{1}{x}$

$f(s,t) = \dfrac{y}{x} = s^{\frac{2}{3}}t^{-\frac{1}{2}}\log t,\quad \dfrac{\partial f}{\partial s} = \dfrac{2}{3}s^{-\frac{1}{3}}t^{-\frac{1}{2}}\log t,\quad \dfrac{\partial f}{\partial t} = -\dfrac{1}{2}s^{\frac{2}{3}}t^{-\frac{3}{2}}\log t + s^{\frac{2}{3}}t^{-\frac{1}{2}}\dfrac{1}{t}$

レポート 10 解答 2 20 点

問 2. (1)

$$
\begin{aligned}
x^2+3xy+2y^2 &= 1\\
2x+3y+3xy'+4yy' &= 0\\
2x+3y+(3x+4y)y' &= 0\\
(3x+4y)y' &= -(2x+3y)\\
y' &= \boxed{-\frac{2x+3y}{3x+4y}}
\end{aligned}
$$

$$
\begin{aligned}
2+3y'+(3+4y')y'+(3x+4y)y'' &= 0\\
2+6y'+4(y')^2+(3x+4y)y'' &= 0\\
(3x+4y)y'' &= -\{2+6y'+4(y')^2\}\\
&= -\left\{2-6\frac{2x+3y}{3x+4y}+4\left(-\frac{2x+3y}{3x+4y}\right)^2\right\}\\
&= \frac{-2(3x+4y)^2+6(2x+3y)(3x+4y)-4(2x+3y)^2}{(3x+4y)^2}\\
&= \frac{2x^2+6xy+4y^2}{(3x+4y)^2}\\
&= \frac{2}{(3x+4y)^2}\\
y'' &= \boxed{\frac{2}{(3x+4y)^3}}
\end{aligned}
$$

(2)

$$
\begin{aligned}
xe^x+ye^y &= 1\\
e^x+xe^x+y'e^y+ye^yy' &= 0\\
(1+x)e^x+(1+y)e^yy' &= 0\\
(1+y)e^yy' &= -(1+x)e^x\\
y' &= -\frac{(1+x)e^x}{(1+y)e^y} = \boxed{-\frac{1+x}{1+y}e^{x-y}}
\end{aligned}
$$

$$
\begin{aligned}
e^x+(1+x)e^x+y'e^yy'+(1+y)e^yy'y'+(1+y)e^yy'' &= 0\\
(2+x)e^x+(2+y)(y')^2e^y+(1+y)e^yy'' &= 0
\end{aligned}
$$

$$
\begin{aligned}
(1+y)e^yy'' &= -\{(2+x)e^x+(2+y)(y')^2e^y\} = -\left\{(2+x)e^x+(2+y)\left(-\frac{1+x}{1+y}e^{x-y}\right)^2e^y\right\}\\
&= -(2+x)e^x-\frac{(2+y)(1+x)^2}{(1+y)^2}e^{2(x-y)+y}\\
y'' &= \boxed{-\frac{(2+x)}{(1+y)}e^{x-y}-\frac{(2+y)(1+x)^2}{(1+y)^3}e^{2(x-y)}}
\end{aligned}
$$

レポート 11 解答 7 点

(1) $\displaystyle \iint_D x^2y^3\,dS = \int_1^2\int_{-1}^2 x^2y^3\,dy\,dx = \int_1^2\left[x^2\frac{y^4}{4}\right]_{-1}^2 dx = \int_1^2 x^2\frac{2^4-(-1)^4}{4}\,dx$

$\displaystyle = \int_1^2 \frac{15}{4}x^2\,dx = \frac{15}{4}\left[\frac{x^3}{3}\right]_1^2 = \frac{15}{4}\frac{2^3-1^3}{3} = \boxed{\frac{35}{4}}$

(2) $\displaystyle \iint_D xy\,dS = \int_0^1\int_0^{x^2} xy\,dy\,dx = \int_0^1\left[x\frac{y^2}{2}\right]_0^{x^2} dx = \int_0^1 x\frac{(x^2)^2-0}{2}\,dx$

$\displaystyle = \int_0^1 \frac{1}{2}x^5\,dx = \frac{1}{2}\left[\frac{x^6}{6}\right]_0^1 = \boxed{\frac{1}{12}}$

(3) $\displaystyle \iint_D \sqrt{x^2-y^2}\,dS = \int_0^1\int_0^x \sqrt{x^2-y^2}\,dy\,dx = \int_0^1\left[\frac{1}{2}\left(y\sqrt{x^2-y^2}+x^2\,\mathrm{Sin}^{-1}\frac{y}{x}\right)\right]_0^x dx$

$\displaystyle = \frac{1}{2}\int_0^1 (x\sqrt{0}+x^2\,\mathrm{Sin}^{-1}1)-(0\sqrt{x^2}+x^2\,\mathrm{Sin}^{-1}0)\,dx = \frac{1}{2}\int_0^1\frac{\pi}{2}x^2\,dx = \frac{\pi}{4}\left[\frac{x^3}{3}\right]_0^1 = \boxed{\frac{\pi}{12}}$

(4) $\displaystyle \iint_D y\,dS = \int_{-1}^2\int_{y^2}^{y+2} y\,dx\,dy = \int_{-1}^2 [xy]_{y^2}^{y+2}\,dy = \int_{-1}^2 \left\{(y+2)y-y^2y\right\}\,dy$

$\displaystyle = \int_{-1}^2\left(-y^3+y^2+2y\right)\,dy = \left[-\frac{y^4}{4}+\frac{y^3}{3}+2\frac{y^2}{2}\right]_{-1}^2$

$\displaystyle = \left(-\frac{2^4}{4}+\frac{2^3}{3}+2^2\right)-\left(-\frac{(-1)^4}{4}+\frac{(-1)^3}{3}+(-1)^2\right) = \boxed{\frac{9}{4}}$

(5) $\displaystyle \iint_D (x^2+y^2)\,dS = \int_0^1\int_0^{1-x}(x^2+y^2)\,dy\,dx = \int_0^1\left[x^2y+\frac{y^3}{3}\right]_0^{1-x} dy$

$\displaystyle = \int_0^1\left\{x^2(1-x)+\frac{(1-x)^3}{3}\right\}\,dy = \frac{1}{3}\int_0^1\left\{3x^2-3x^3+(1-3x+3x^2-x^3)\right\}\,dy$

$\displaystyle = \frac{1}{3}\int_0^1(-4x^3+6x^2-3x+1)\,dx = \frac{1}{3}\left[-x^4+2x^3-3\frac{x^2}{2}+x\right]_0^1 = \frac{1}{3}\left(-1+2-3\frac{1}{2}+1\right) = \boxed{\frac{1}{6}}$

(6) $\displaystyle \iint_D \cos(x+2y)\,dS = \int_0^{\frac{\pi}{2}}\int_0^x \cos(x+2y)\,dy\,dx = \int_0^{\frac{\pi}{2}}\left[\frac{1}{2}\sin(x+2y)\right]_0^x dx$

$\displaystyle = \frac{1}{2}\int_0^{\frac{\pi}{2}}(\sin 3x-\sin x)\,dy = \frac{1}{2}\left[-\frac{1}{3}\cos 3x+\cos x\right]_0^{\frac{\pi}{2}}$

$\displaystyle = \frac{1}{2}\left\{-\frac{1}{3}\cos\frac{3}{2}\pi+\cos\frac{1}{2}\pi-(-\frac{1}{3}\cos 0+\cos 0)\right\} = \frac{1}{2}\left(-0+0+\frac{1}{3}-1\right) = \boxed{-\frac{1}{3}}$

注 $\displaystyle \cos\frac{3}{2}\pi = 0,\qquad \cos\frac{1}{2}\pi = 0,\qquad \cos 0 = 1$

(7) $\displaystyle \iint_D x\,dS = \int_0^1\int_{1-x}^{\sqrt{1-x^2}} x\,dy\,dx = \int_0^1 [xy]_{1-x}^{\sqrt{1-x^2}}\,dx = \int_0^1\left\{x\sqrt{1-x^2}-x(1-x)\right\}\,dx$

$\displaystyle = \left[-\frac{1}{3}\sqrt{1-x^2}^3-\frac{x^2}{2}+\frac{x^3}{3}\right]_0^1 = -0-\frac{1}{2}+\frac{1}{3}-(-\frac{1}{3}-0+0) = \boxed{\frac{1}{6}}$

注 $\displaystyle \int x\sqrt{1-x^2}\,dx = \int xt^{\frac{1}{2}}\frac{1}{(-2x)}\,dt = -\frac{1}{3}t^{\frac{3}{2}}+C = -\frac{1}{3}\sqrt{1-x^2}^3+C\quad (t=1-x^2)$

レポート 12　解答 8 点

(1) $\dfrac{dy}{dx} = xy, \qquad \dfrac{1}{y}\,dy = x\,dx, \qquad \displaystyle\int \frac{1}{y}\,dy = \int x\,dx,$

$\log|y| = \dfrac{x^2}{2} + C, \qquad y = \pm e^{\frac{x^2}{2}+C}, \qquad \boxed{y = Ae^{\frac{x^2}{2}}}$　注　$A = \pm e^C$

(2) $y\dfrac{dy}{dx} = -x^2, \qquad y\,dy = -x^2\,dx, \qquad \displaystyle\int y\,dy = -\int x^2\,dx,$

$\dfrac{y^2}{2} = -\dfrac{x^3}{3} + C, \qquad y^2 = -\dfrac{2}{3}x^3 + 2C, \qquad \boxed{y = \pm\sqrt{C - \dfrac{2}{3}x^3}}$　注　$2C$ を新たに C とする

(3) $x^2\dfrac{dy}{dx} = -y, \qquad \dfrac{1}{y}\,dy = -\dfrac{1}{x^2}\,dx, \qquad \displaystyle\int \frac{1}{y}\,dy = -\int x^{-2}\,dx,$

$\log|y| = \dfrac{1}{x} + C, \qquad y = \pm e^{\frac{1}{x}+C}, \qquad \boxed{y = Ae^{\frac{1}{x}}}$　注　$A = \pm e^C$

(4) $\dfrac{dy}{dx} = y^2x, \qquad \dfrac{1}{y^2}\,dy = x\,dx, \qquad \displaystyle\int y^{-2}\,dy = \int x\,dx,$

$-\dfrac{1}{y} = \dfrac{x^2}{2} + C, \qquad y = \dfrac{2}{-x^2 - 2C}, \qquad \boxed{y = -\dfrac{2}{x^2 + C}}$　注　$C = 2C$

(5) $(x+1)\dfrac{dy}{dx} = y + 1, \qquad \dfrac{1}{y+1}\,dy = \dfrac{1}{x+1}\,dx, \qquad \displaystyle\int \frac{1}{y+1}\,dy = \int \frac{1}{x+1}\,dx,$

$\log|y+1| = \log|x+1| + C, \quad \log\left|\dfrac{y+1}{x+1}\right| = C, \quad y + 1 = A(x+1), \quad \boxed{y = A(x+1) - 1}$

注　$A = \pm e^C$

(6) $x^3\dfrac{dy}{dx} = y^2, \qquad \dfrac{1}{y^2}\,dy = \dfrac{1}{x^3}\,dx, \qquad \displaystyle\int \frac{1}{y^2}\,dy = \int \frac{1}{x^3}\,dx,$

$-\dfrac{1}{y} = -\dfrac{1}{2x^2} + C, \qquad y = \dfrac{2x^2}{1 - 2Cx^2}, \qquad y = \dfrac{2x^2}{1 - Cx^2}$　注　$C = 2C$

$-1 = \dfrac{2 \cdot 1^2}{1 - C1^2}, \qquad C - 1 = 2, \qquad C = 3, \qquad \boxed{y = \dfrac{2x^2}{1 - 3x^2}}$

(7) $x\dfrac{dy}{dx} = -y, \qquad \dfrac{1}{y}\,dy = -\dfrac{1}{x}\,dx, \qquad \displaystyle\int \frac{1}{y}\,dy = -\int \frac{1}{x}\,dx,$

$\log|y| = -\log|x| + C, \qquad y = \pm e^{-\log|x|+C}, \qquad y = A\dfrac{1}{|x|} = \dfrac{A}{x}$　注　$A = \pm A$

$1 = \dfrac{1}{A}, \qquad A = 1, \qquad \boxed{y = \dfrac{1}{x}}$

(8) $\dfrac{dy}{dx} = (y^2 + 4)x, \qquad \dfrac{1}{y^2 + 4}\,dy = x\,dx, \qquad \displaystyle\int \frac{1}{y^2 + 4}\,dy = \int x\,dx,$

$\dfrac{1}{2}\,\mathrm{Tan}^{-1}\,\dfrac{y}{2} = \dfrac{x^2}{2} + C, \qquad \mathrm{Tan}^{-1}\,\dfrac{y}{2} = x^2 + 2C, \qquad \dfrac{y}{2} = \tan(x^2 + 2C), \qquad y = 2\tan(x^2 + C)$

注　$C = 2C$

$2 = 2\tan(0^2 + C), \qquad 1 = \tan C, \qquad C = \dfrac{\pi}{4}, \qquad \boxed{y = 2\tan\left(x^2 + \dfrac{\pi}{4}\right)}$