

微積分半期演習レポート解答

レポート 3 解答 9 点

$$(1) y' = \boxed{2^x \log 2}$$

$$(2) y' = \boxed{\frac{1}{x \log 3}}$$

$$(3) y' = \boxed{2e^{2x+3}}$$

$$(4) y' = \frac{(x^2 - 2x + 3)'}{x^2 - 2x + 3} = \boxed{\frac{2x - 2}{x^2 - 2x + 3}}$$

$$(5) y' = (2 - x)'e^{x^2} + (2 - x)(e^{x^2})' = -e^{x^2} + (2 - x)e^{x^2}(x^2)'$$

$$= -e^{x^2} + (2 - x)e^{x^2}(2x) = \boxed{(-1 + 4x - 2x^2)e^{x^2}}$$

$$\text{注)} \quad (fg)' = f'g + fg'$$

$$(6) y' = (3^x)' \log_2 x + 3^x (\log_2 x)' = \boxed{3^x \log 3 \log_2 x + \frac{3^x}{x \log 2}}$$

$$(7) y' = (\log |x - a| - \log |x + a|)' = \frac{1}{x - a} - \frac{1}{x + a} = \frac{(x + a) - (x - a)}{(x - a)(x + a)} = \boxed{\frac{2a}{x^2 - a^2}}$$

$$\text{注)} \quad \log \frac{a}{b} = \log a - \log b$$

$$(8) y' = \frac{(x + \sqrt{x^2 + A})'}{x + \sqrt{x^2 + A}} = \frac{1 + \frac{(x^2 + A)'}{2\sqrt{x^2 + A}}}{x + \sqrt{x^2 + A}} = \frac{1 + \frac{2x}{2\sqrt{x^2 + A}}}{x + \sqrt{x^2 + A}}$$

$$= \frac{\frac{\sqrt{x^2 + A} + x}{\sqrt{x^2 + A}}}{x + \sqrt{x^2 + A}} = \frac{\sqrt{x^2 + A} + x}{(x + \sqrt{x^2 + A})(\sqrt{x^2 + A})} = \boxed{\frac{1}{\sqrt{x^2 + A}}}$$

$$\text{注)} \quad (\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$(9) y' = (x)' \sqrt{x^2 + A} + x(\sqrt{x^2 + A})' + A(\log(x + \sqrt{x^2 + A}))'$$

$$= \sqrt{x^2 + A} + x \frac{2x}{2\sqrt{x^2 + A}} + \frac{A}{\sqrt{x^2 + A}} = \frac{(\sqrt{x^2 + A})^2 + x^2 + A}{\sqrt{x^2 + A}}$$

$$= \frac{2(x^2 + A)}{\sqrt{x^2 + A}} = \boxed{2\sqrt{x^2 + A}}$$

$$\text{注)} \quad (\sqrt{a})^2 = a, \quad \frac{a}{\sqrt{a}} = \sqrt{a}$$

レポート 4 解答 8 点

$$(1) y' = (\cos(2x - 3))(2x - 3)' = \boxed{2 \cos(2x - 3)}$$

$$(2) y' = (-\sin x^3)(x^3)' = \boxed{-3x^2 \sin(x^3)}$$

$$(3) y' = 4 \sin^3 x (\sin x)' = \boxed{4 \sin^3 x \cos x}$$

$$\text{注 } \sin^n x = (\sin x)^n$$

$$(4) y' = \frac{(\cos x)' \sin x - \cos x (\sin x)'}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \boxed{-\frac{1}{\sin^2 x}}$$

$$\text{注 } \frac{f}{g} = \frac{f'g - fg'}{g^2}, \quad \sin^2 x + \cos^2 x = 1$$

$$(5) y' = \frac{(x^2 - x + 1)'}{\cos^2(x^2 - x + 1)} = \boxed{\frac{2x - 1}{\cos^2(x^2 - x + 1)}}$$

$$(6) y' = (\sin^2 x)' \cos 3x + \sin^2 x (\cos 3x)' \\ = 2 \sin x (\sin x)' \cos 3x + \sin^2 x (-\sin 3x)(3x)' = \boxed{2 \sin x \cos x \cos 3x - 3 \sin^2 x \sin 3x}$$

$$(7) y' = \frac{(\cos x)'}{\cos x} = -\frac{\sin x}{\cos x} = \boxed{-\tan x}$$

$$(8) y' = (e^{2x})' \sin 3x + e^{2x} (\sin 3x)' = e^{2x} (2x)' \sin 3x + e^{2x} \cos 3x (3x)' = \boxed{2e^{2x} \sin 3x + 3e^{2x} \cos 3x}$$

レポート 5 解答 No. 1

$$(1) y = \frac{1}{3}x^3 + x^2 - 3x + 3$$

$$y' = x^2 + 2x - 3 = (x+3)(x-1)$$

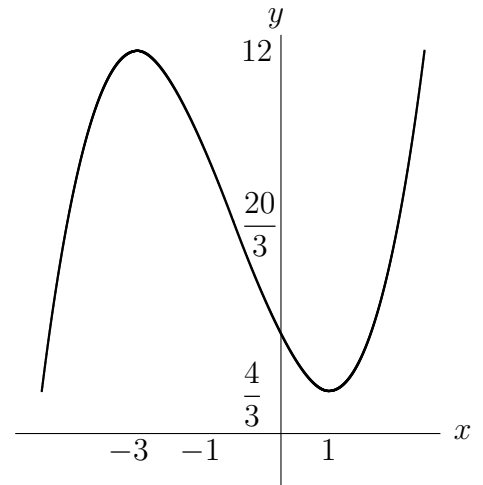
$$y'' = 2x + 2 = 2(x+1)$$

$$y' = 0 \text{ となるのは } x = -3, 1$$

$$y'' = 0 \text{ となるのは } x = -1$$

x	...	-3	...	-1	...	1	...
y'	+	0	-	-	-	0	+
y''	-	-	-	0	+	+	+
y	\curvearrowright	12 極大	\curvearrowleft	$\frac{20}{3}$ 変曲点	\curvearrowleft	$\frac{4}{3}$ 極小	\curvearrowright

右のグラフで、 y 軸は x 軸の $\frac{2}{3}$ 倍



$$(2) y = x^4 - 6x^2 + 1$$

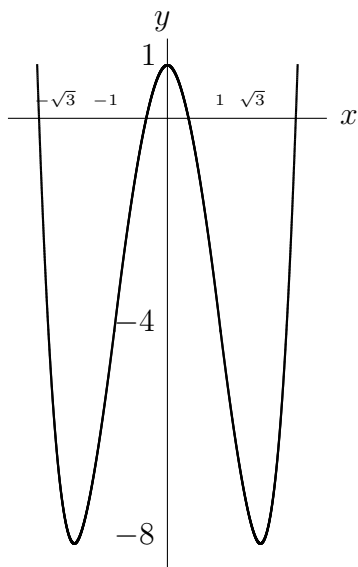
$$y' = 4x^3 - 12x = 4x(x^2 - 3)$$

$$y'' = 12x^2 - 12 = 12(x^2 - 1)$$

$$y' = 0 \text{ となるのは } x = 0, \pm\sqrt{3}$$

$$y'' = 0 \text{ となるのは } x = \pm 1$$

x	...	$-\sqrt{3}$...	-1	...	0	...	1	...	$\sqrt{3}$...
y'	-	0	+	+	+	0	-	-	-	0	+
y''	+	+	+	0	-	-	-	0	+	+	+
y	\curvearrowleft	-8 極小	\curvearrowright	-4 変曲点	\curvearrowright	1 極大	\curvearrowleft	-4 変曲点	\curvearrowleft	-8 極小	\curvearrowright



レポート 5 解答 No. 2

$$(3) y = \frac{1}{x} + x$$

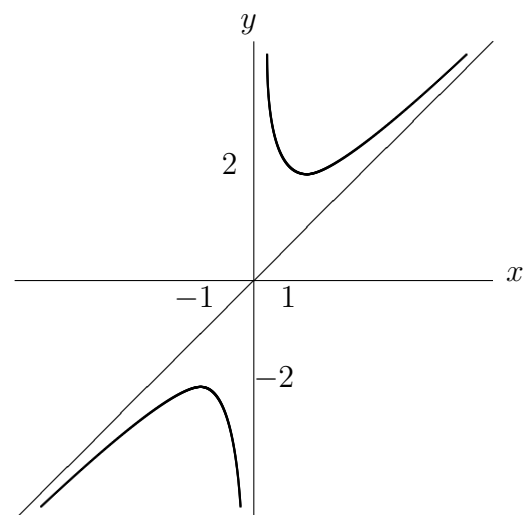
$$y' = -\frac{1}{x^2} + 1$$

$$y'' = \frac{2}{x^3}$$

$$y' = 0 \text{ となるのは } x = \pm 1$$

$$y'' = 0 \text{ となる } x \text{ はない}$$

x	\dots	-1	\dots	0	\dots	1	\dots
y'	$+$	0	$-$	\times	$-$	0	$+$
y''	$-$	$-$	$-$	\times	$+$	$+$	$+$
y	\curvearrowright	-2 極大	\curvearrowleft	\times	\curvearrowright	2 極小	\curvearrowleft



$$(4) y = xe^{-2x}$$

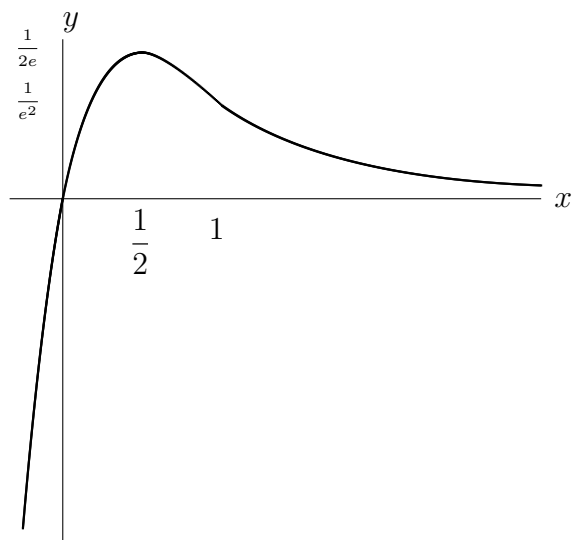
$$y' = (1 - 2x)e^{-2x}$$

$$y'' = 4(x - 1)e^{-2x}$$

$$y' = 0 \text{ となるのは } x = \frac{1}{2}$$

$$y'' = 0 \text{ となるのは } x = 1$$

x	\dots	$\frac{1}{2}$	\dots	1	\dots
y'	$+$	0	$-$	$-$	$-$
y''	$-$	$-$	$-$	0	$+$
y	\curvearrowright	$\frac{1}{2e}$ 極大	\curvearrowleft	$\frac{1}{e^2}$ 変曲点	\curvearrowright



右のグラフで、 y 軸は x 軸の 5 倍

レポート 5 解答 No. 3

$$(5) y = e^{-x^2}$$

$$y' = -2xe^{-x^2}$$

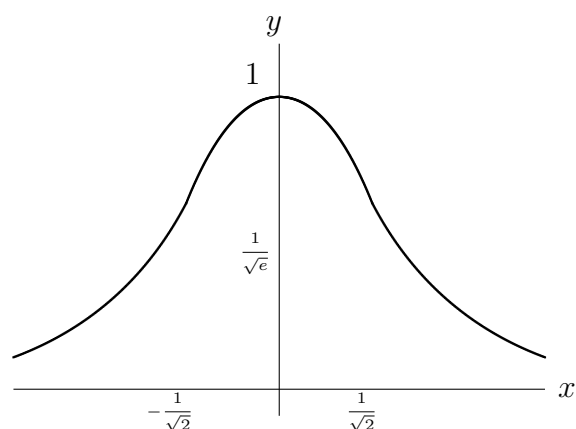
$$y'' = 2(2x^2 - 1)e^{-x^2}$$

$$y' = 0 \text{ となるのは } x = 0$$

$$y'' = 0 \text{ となるのは } x = \pm \frac{1}{\sqrt{2}}$$

x	\cdots	$-\frac{1}{\sqrt{2}}$	\cdots	0	\cdots	$\frac{1}{\sqrt{2}}$	\cdots
y'	+	+	+	0	-	-	-
y''	+	0	-	-	-	0	+
y	\nearrow	$\frac{1}{\sqrt{e}}$ 変曲点	\nearrow	1 極大	\searrow	$\frac{1}{\sqrt{e}}$ 変曲点	\searrow

右のグラフで、 y 軸は x 軸の 2 倍



$$(6) y = \log(1 + x^2)$$

$$y' = \frac{2x}{1 + x^2}$$

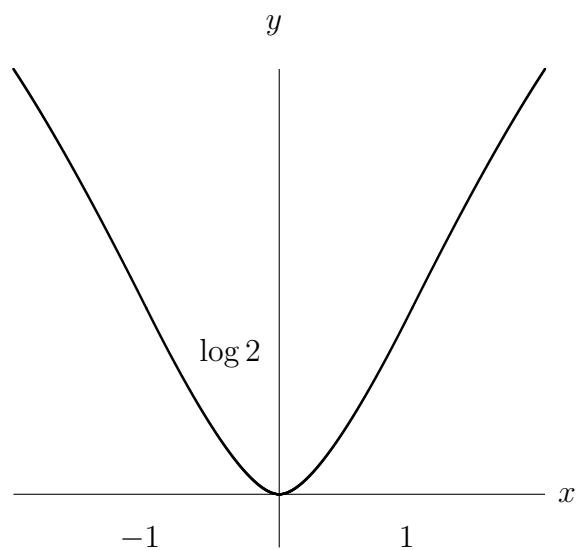
$$y'' = \frac{2(1 - x^2)}{1 + x^2}$$

$$y' = 0 \text{ となるのは } x = 0$$

$$y'' = 0 \text{ となるのは } x = \pm 1$$

x	\cdots	-1	\cdots	0	\cdots	1	\cdots
y'	-	-	-	0	+	+	+
y''	-	0	+	+	+	0	-
y	\searrow	$\log 2$ 変曲点	\searrow	0 極小	\nearrow	$\log 2$ 変曲点	\nearrow

右のグラフで、 y 軸は x 軸の 2 倍



レポート 5 解答 No. 4

$$(7) y = x^2 \log x$$

$$y' = x(2 \log x + 1)$$

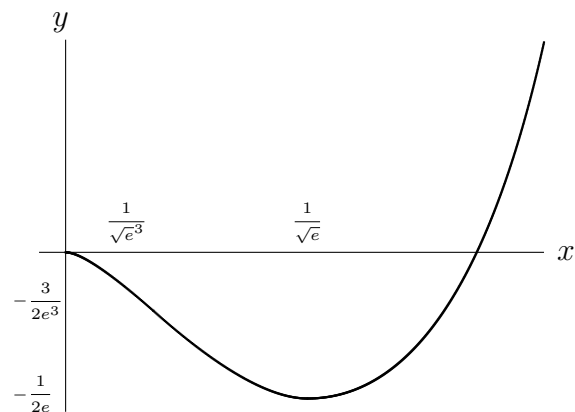
$$y'' = 2 \log x + 3$$

$$y' = 0 \text{ となるのは } x = 0, \frac{1}{\sqrt{e}}$$

$$y'' = 0 \text{ となるのは } x = \frac{1}{\sqrt{e^3}}$$

x	0	...	$\frac{1}{\sqrt{e^3}}$...	$\frac{1}{\sqrt{e}}$...
y'	0	-	-	-	0	+
y''	\times	-	0	+	+	+
y	0	\curvearrowright	$-\frac{3}{2e^3}$ 変曲点	\curvearrowleft	$-\frac{1}{2e}$ 極小	\curvearrowright

右のグラフで、 y 軸は x 軸の 2 倍



$$(8) y = x + \sin x \quad (0 \leq x \leq 2\pi)$$

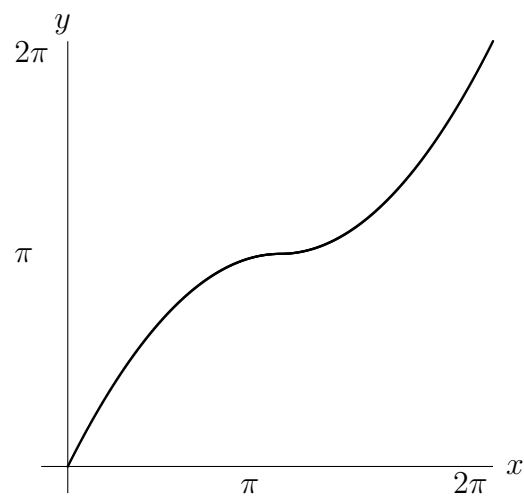
$$y' = 1 + \cos x \geq 0$$

$$y'' = -\sin x$$

$$y' = 0 \text{ となるのは } x = \pi$$

$$y'' = 0 \text{ となるのは } x = 0, \pi, 2\pi$$

x	0	...	π	...	2π
y'	+	+	0	+	+
y''	0	-	0	+	0
y	0	\curvearrowright	π 変曲点	\curvearrowright	2π



レポート 6 解答

$$(1) \int \frac{1}{x^2 + 2x} dx \int \frac{1}{(x+1)^2 - 1} dx = \boxed{\frac{1}{2} \log \left| \frac{x}{x+2} \right| + C}$$

$$(2) \int \frac{1}{2x^2 + 4x + 4} dx = \frac{1}{2} \int \frac{1}{(x+1)^2 + 1} dx = \boxed{\frac{1}{2} \text{Tan}^{-1}(x+1) + C}$$

$$(3) \int \frac{1}{\sqrt{-4x^2 + 8x - 3}} dx = \frac{1}{2} \int \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 - (x-1)^2}} = \boxed{\frac{1}{2} \text{Sin}^{-1} 2(x-1) + C}$$

$$(4) \int \frac{1}{\sqrt{4x^2 - 8x + 3}} dx = \frac{1}{2} \int \frac{1}{\sqrt{(x-1)^2 - \frac{1}{4}}} dx = \frac{1}{2} \log \left| (x-1) + \sqrt{(x-1)^2 - \frac{1}{4}} \right| + C$$

$$= \boxed{\frac{1}{2} \log \left| 2x - 2 + \sqrt{4x^2 - 8x + 3} \right| + C}$$

注 $\frac{1}{2} \log \frac{|f(x)|}{2} + C = \frac{1}{2} \log |f(x)| - \frac{1}{2} \log 2 + C$ であり、

$-\frac{1}{2} \log 2 + C$ を新たに C としている。

$$(5) \int \sqrt{-x^2 + 4x - 1} dx = \int \sqrt{\sqrt{3}^2 - (x-2)^2} dx$$

$$= \boxed{\frac{1}{2} \left\{ (x-2)\sqrt{-x^2 + 4x - 1} + 3 \text{Sin}^{-1} \frac{x-2}{\sqrt{3}} \right\} + C}$$

$$(6) \int \sqrt{x^2 - 4x + 1} dx = \int \sqrt{(x-2)^2 - 3} dx$$

$$= \boxed{\frac{1}{2} \left\{ (x-2)\sqrt{x^2 - 4x + 1} - 3 \log \left| x - 2 + \sqrt{x^2 - 4x + 1} \right| \right\} + C}$$

レポート 7 解答

$$\text{問 1. (1)} = \int t^3(x-1) \frac{1}{2x-2} dt = \frac{1}{2} \int t^3 dt = \frac{1}{2} \frac{t^4}{4} + C = \boxed{\frac{(x^2 - 2x - 1)^4}{8} + C}$$

$$\text{注 } \frac{dt}{dx} = (x^2 - 2x - 1)' = 2x - 2, \quad dx = \frac{1}{(2x-2)} dt$$

$$(2) = \int \frac{\cos x}{t} \frac{1}{\cos x} dt = \int \frac{1}{t} dt = \log |t| + C = \boxed{\log |\sin x| + C}$$

$$(3) = \int (t^2 - 1)t 2t dt = \int 2t^4 - 2t^2 dt = \frac{2}{5}t^5 - \frac{2}{3}t^3 + C = \boxed{\frac{2}{5}\sqrt{x+1}^5 - \frac{2}{3}\sqrt{x+1}^3 + C}$$

$$\text{注 } \frac{dx}{dt} = (t^2 - 1)' = 2t, \quad dx = 2t dt$$

$$(4) = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{\sin^2 x} dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{1 - \cos^2 x} dx = \int_{\frac{1}{2}}^0 \frac{\sin x}{1 - t^2} \frac{1}{(-\sin x)} dt = \int_{\frac{1}{2}}^0 \frac{1}{t^2 - 1} dt$$

$$= \left[\frac{1}{2} \log \left| \frac{t-1}{t+1} \right| \right]_{\frac{1}{2}}^0 = \frac{1}{2} (\log 1 - \log \frac{1}{2}) = \boxed{\frac{1}{2} \log 3}$$

$$\text{注 } \cos^2 x + \sin^2 x = 1, \quad \cos \frac{\pi}{2} = 0, \quad \cos \frac{\pi}{3} = \frac{1}{2}, \quad \log 1 = 0, \quad \log \frac{1}{a} = -\log a$$

$$(5) = \int_0^1 \frac{t^2}{x} dt = \int_0^1 t^2 dt = \left[\frac{t^3}{3} \right]_0^1 = \boxed{\frac{1}{3}}$$

$$\text{注 } \frac{dt}{dx} = (\log x)' = \frac{1}{x}, \quad dx = x dt, \quad \log e = 1, \quad \log 1 = 0$$

$$(6) = \int_1^{\sqrt{3}} \frac{e^x}{t^2 + 1} \frac{1}{e^x} dt = \int_1^{\sqrt{3}} \frac{1}{1 + t^2} dt = [\text{Tan}^{-1} t]_1^{\sqrt{3}} = \text{Tan}^{-1} \sqrt{3} - \text{Tan}^{-1} 1 = \frac{\pi}{3} - \frac{\pi}{4} = \boxed{\frac{\pi}{12}}$$

$$\text{注 } e^0 = 1, \quad e^{\log a} = a$$

$$\text{問 2. (1)} \int \frac{1}{\sin x} dx = \int \frac{1}{\left(\frac{2t}{1+t^2}\right) (1+t^2)} dt = \int \frac{1}{t} dt = \log |t| + C = \boxed{\log \left| \tan \frac{x}{2} \right| + C}$$

$$(2) \int \frac{1}{2 \cos x + 3} dx = \int \frac{1}{\left(2 \frac{1-t^2}{1+t^2} + 3\right) (1+t^2)} dt = \int \frac{2}{t^2 + 5} dt = \frac{2}{\sqrt{5}} \text{Tan}^{-1} \frac{t}{\sqrt{5}} + C$$

$$= \boxed{\frac{2}{\sqrt{5}} \text{Tan}^{-1} \left(\frac{1}{\sqrt{5}} \tan \frac{x}{2} \right) + C}$$

$$(3) \int \frac{1}{2 \sin x + 3} dx = \int \frac{1}{\left(2 \frac{2t}{1+t^2} + 3\right) (1+t^2)} dt = \int \frac{2}{3t^2 + 4t + 3} dt = \frac{2}{3} \int \frac{1}{\left(t + \frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2} dt$$

$$= \frac{2}{3} \frac{1}{\frac{\sqrt{5}}{3}} \text{Tan}^{-1} \frac{t + \frac{2}{3}}{\frac{\sqrt{5}}{3}} + C = \boxed{\frac{2}{\sqrt{5}} \text{Tan}^{-1} \left(\frac{3 \tan \frac{x}{2} + 2}{\sqrt{5}} \right) + C}$$

レポート 8 解答 No.1 10 点

$$\begin{aligned} \text{問 1. (1)} &= \int \left(\frac{x^3}{3} \right)' \log x \, dx = \frac{x^3}{3} \log x - \int \frac{x^3}{3} (\log x)' \, dx \\ &= \frac{x^3 \log x}{3} - \int \frac{x^3}{3} \frac{1}{x} \, dx = \frac{x^3 \log x}{3} - \int \frac{x^2}{3} \, dx = \boxed{\frac{x^3 \log x}{3} - \frac{x^3}{9} + C}, \quad \text{注 } \int x^2 \, dx = \frac{x^3}{3} + C \end{aligned}$$

$$\begin{aligned} (2) &= \int x^2 (\sin x)' \, dx = x^2 \sin x - \int (x^2)' \sin x \, dx = x^2 \sin x - 2 \int x \sin x \, dx \\ &= x^2 \sin x - 2 \int x (-\cos x)' \, dx = x^2 \sin x - 2 \left(-x \cos x + \int (x)' \cos x \, dx \right) \\ &= x^2 \sin x + 2x \cos x - 2 \int \cos x \, dx = \boxed{x^2 \sin x + 2x \cos x - 2 \sin x + C}, \end{aligned}$$

$$\text{注 } \int \cos x \, dx = \sin x + C, \quad \int \sin x \, dx = -\cos x + C$$

$$(3) = \int x \left(\frac{e^{3x}}{3} \right)' \, dx = x \frac{e^{3x}}{3} - \int (x)' \frac{e^{3x}}{3} \, dx = x \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} \, dx = \boxed{x \frac{e^{3x}}{3} - \frac{e^{3x}}{9} + C}$$

$$\text{注 } \int e^{3x} \, dx = \frac{e^{3x}}{3} + C$$

$$\begin{aligned} (4) &= \int_1^e \left(\frac{x^2}{2} \right)' \log x \, dx = \left[\frac{x^2}{2} \log x \right]_1^e - \int_1^e \frac{x^2}{2} (\log x)' \, dx \\ &= \frac{e^2}{2} - \int_1^e \frac{x^2}{2} \frac{1}{x} \, dx = \frac{e^2}{2} - \int_1^e \frac{x}{2} \, dx = \frac{e^2}{2} - \left[\frac{x^2}{4} \right]_1^e = \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} = \boxed{\frac{e^2 + 1}{4}} \end{aligned}$$

$$\text{注 } \int x \, dx = \frac{x^2}{2} + C, \quad \log e = 1, \quad \log 1 = 0, \quad \frac{a}{2} - \frac{a}{4} = \frac{2a - a}{4} = \frac{a}{4}$$

$$\begin{aligned} (5) &= \int_0^{\frac{\pi}{6}} x \left(-\frac{\cos 2x}{2} \right)' \, dx = \left[-x \frac{\cos 2x}{2} \right]_0^{\frac{\pi}{6}} + \frac{1}{2} \int_0^{\frac{\pi}{6}} (x)' \cos 2x \, dx = -\frac{1}{2} \frac{\pi}{6} \cos \frac{\pi}{3} + \frac{1}{2} \int_0^{\frac{\pi}{6}} \cos 2x \, dx \\ &= -\frac{\pi}{24} + \frac{1}{2} \left[\frac{\sin 2x}{2} \right]_0^{\frac{\pi}{6}} = -\frac{\pi}{24} + \frac{1}{4} \sin \frac{\pi}{3} = \boxed{-\frac{\pi}{24} + \frac{\sqrt{3}}{8}} \end{aligned}$$

$$\text{注 } \int \sin 2x \, dx = -\frac{\cos 2x}{2} + C, \quad \cos 0 = 1, \quad \cos \frac{\pi}{3} = \frac{1}{2}, \quad \sin 0 = 0, \quad \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} (6) &= \int_0^1 x^3 (e^x)' \, dx = [x^3 e^x]_0^1 - \int_0^1 (x^3)' e^x \, dx = e - 3 \int_0^1 x^2 (e^x)' \, dx \\ &= e - 3 \left([x^2 e^x]_0^1 - \int_0^1 (x^2)' e^x \, dx \right) = e - 3 \left(e - \int_0^1 2x e^x \, dx \right) = -2e + 6 \int_0^1 x (e^x)' \, dx \\ &= -2e + 6 \left([x e^x]_0^1 - \int_0^1 (x)' e^x \, dx \right) = -2e + 6 (e - [e^x]_0^1) = -2e + 6e - 6(e - 1) = \boxed{-2e + 6} \end{aligned}$$

$$\text{注 } \int e^x \, dx = e^x + C, \quad e^0 = 1, \quad e^1 = e$$

レポート 8 解答 No.2

問 2. (1) 部分分数分解して、 $\frac{x}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$

$$A = \frac{x}{(x-2)(x+1)}(x-2) \Big|_{x=2} = \frac{2}{3},$$

$$B = \frac{x}{(x-2)(x+1)}(x+1) \Big|_{x=-1} = \frac{1}{3}$$

$$\begin{aligned} \int \frac{x}{(x-2)(x+1)} dx &= \int \left\{ \frac{\frac{2}{3}}{x-2} + \frac{\frac{1}{3}}{x+1} \right\} dx \\ &= \frac{2}{3} \log |x-2| + \frac{1}{3} \log |x+1| + C \\ &= \boxed{\log \left| \sqrt[3]{(x-2)^2(x+1)} \right| + C} \end{aligned}$$

(2) 部分分数分解して、 $\frac{x+3}{(x-1)(x+1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2}$

$$A = \frac{x+3}{(x-1)(x+1)(x+2)}(x-1) \Big|_{x=1} = \frac{2}{3},$$

$$B = \frac{x+3}{(x-1)(x+1)(x+2)}(x+1) \Big|_{x=-1} = -1,$$

$$C = \frac{x+3}{(x-1)(x+1)(x+2)}(x+2) \Big|_{x=-2} = \frac{1}{3},$$

$$\begin{aligned} \int \frac{x+3}{(x-1)(x+1)(x+2)} dx &= \int \left\{ \frac{\frac{2}{3}}{x-1} - \frac{1}{x+1} + \frac{\frac{1}{3}}{x+2} \right\} dx \\ &= \frac{2}{3} \log |x-1| - \log |x+1| + \frac{1}{3} \log |x+2| + C \\ &= \boxed{\log \left| \frac{\sqrt[3]{(x-1)^2(x+2)}}{x+1} \right| + C} \end{aligned}$$

レポート 8 解答 No.3

(3) 部分分数分解して、 $\frac{x^2 + x + 1}{(x-1)x^2} = \frac{A}{x-1} + \frac{B_2}{x^2} + \frac{B_1}{x}$

$$\begin{aligned} A &= \frac{x^2 + x + 1}{(x-1)x^2}(x-1) \Big|_{x=1} = 3, \\ B_2 &= \frac{x^2 + x + 1}{(x-1)x^2}x^2 \Big|_{x=0} = -1, \\ B_1 &= \frac{1}{1!} \frac{d}{dx} \left\{ \frac{x^2 + x + 1}{(x-1)x^2} x^2 \right\} \Big|_{x=0} = \frac{(x^2 + x + 1)'(x-1) - (x^2 + x + 1)(x-1)'}{(x-1)^2} \Big|_{x=0} \\ &= \frac{x^2 - 2x - 2}{(x-1)^2} \Big|_{x=0} = -2, \end{aligned}$$

$$\begin{aligned} \int \frac{x^2 + x + 1}{(x-1)x^2} dx &= \int \left\{ \frac{3}{x-1} - \frac{1}{x^2} - \frac{2}{x} \right\} dx \\ &= 3 \log |x-1| + \frac{1}{x} - 2 \log |x| + C \\ &= \boxed{\frac{1}{x} + \log \left| \frac{(x-1)^3}{x^2} \right| + C} \end{aligned}$$

(4) 部分分数分解して、 $\frac{x+1}{x(x+2)^3} = \frac{A}{x} + \frac{B_3}{(x+2)^3} + \frac{B_2}{(x+2)^2} + \frac{B_1}{(x+2)}$

$$\begin{aligned} A &= \frac{x+1}{x(x+2)^3}x \Big|_{x=0} = \frac{1}{8}, \\ B_3 &= \frac{x+1}{x(x+2)^3}(x+2)^3 \Big|_{x=-2} = \frac{1}{2}, \\ B_2 &= \frac{1}{1!} \frac{d}{dx} \left\{ \frac{x+1}{x(x+2)^3}(x+2)^3 \right\} \Big|_{x=-2} = \frac{(x+1)'x - (x+1)(x)'}{x^2} \Big|_{x=-2} \\ &= \frac{-1}{x^2} \Big|_{x=-2} = -\frac{1}{4}, \\ B_1 &= \frac{1}{2!} \frac{d^2}{dx^2} \left\{ \frac{x+1}{x(x+2)^3}(x+2)^3 \right\} \Big|_{x=-2} = \frac{1}{2} \frac{d}{dx} \left\{ \frac{-1}{x^2} \right\} \Big|_{x=-2} = \frac{1}{2} \frac{2}{x^3} \Big|_{x=-2} = -\frac{1}{8}, \end{aligned}$$

$$\begin{aligned} \int \frac{x+1}{x(x+2)^3} dx &= \int \left\{ \frac{\frac{1}{8}}{x} + \frac{\frac{1}{2}}{(x+2)^3} + \frac{-\frac{1}{4}}{(x+2)^2} + \frac{-\frac{1}{8}}{(x+2)} \right\} dx \\ &= \frac{1}{8} \log |x| - \frac{1}{4} \frac{1}{(x+2)^2} + \frac{1}{4} \frac{1}{(x+2)} - \frac{1}{8} \log |x+2| + C \\ &= \boxed{\frac{(x+1)}{4(x+2)^2} + \log \sqrt[8]{\left| \frac{x}{x+2} \right|} + C} \end{aligned}$$

レポート 9 解答 No.1 18 点

$$(1) z_x = \boxed{6x^2 - 8xy^3 + 3y^4}, \quad z_y = \boxed{-12x^2y^2 + 12xy^3 - 10y^4}$$

$$z_{xx} = \boxed{12x - 8y^3}, \quad z_{xy} = \boxed{-24xy^2 + 12y^3}, \quad z_{yx} = \boxed{-24xy^2 + 12y^3}, \quad z_{yy} = \boxed{-24x^2y + 36xy^2 - 40y^3}$$

$$(2) z_x = \frac{\partial_x(x^2 + xy)}{2\sqrt{x^2 + xy}} = \boxed{\frac{2x + y}{2\sqrt{x^2 + xy}}}, \quad z_y = \frac{\partial_y(x^2 + xy)}{2\sqrt{x^2 + xy}} = \boxed{\frac{x}{2\sqrt{x^2 + xy}}}$$

注 以下は上の z_x, z_y の計算結果を使用する。

$$z_{xx} = \frac{\partial_x(2x + y)2\sqrt{x^2 + xy} - (2x + y)\partial_x(2\sqrt{x^2 + xy})}{4(\sqrt{x^2 + xy})^2} = \frac{4\sqrt{x^2 + xy} - (2x + y)2\frac{(2x+y)}{2\sqrt{x^2+xy}}}{4(x^2 + xy)}$$

$$= \frac{4(\sqrt{x^2 + xy})^2 - (2x + y)^2}{4(x^2 + xy)\sqrt{x^2 + xy}} = \boxed{-\frac{y^2}{4(x^2 + xy)\sqrt{x^2 + xy}}}$$

$$z_{xy} = \frac{\partial_y(2x + y)2\sqrt{x^2 + xy} - (2x + y)\partial_y(2\sqrt{x^2 + xy})}{4(\sqrt{x^2 + xy})^2} = \frac{2\sqrt{x^2 + xy} - (2x + y)2\frac{x}{2\sqrt{x^2+xy}}}{4(x^2 + xy)}$$

$$= \frac{2(\sqrt{x^2 + xy})^2 - (2x + y)x}{4(x^2 + xy)\sqrt{x^2 + xy}} = \boxed{\frac{xy}{4(x^2 + xy)\sqrt{x^2 + xy}}}$$

$$z_{yx} = \frac{\partial_x(x)2\sqrt{x^2 + xy} - x\partial_x(2\sqrt{x^2 + xy})}{4(\sqrt{x^2 + xy})^2} = \frac{2\sqrt{x^2 + xy} - x2\frac{(2x+y)}{2\sqrt{x^2+xy}}}{4(x^2 + xy)}$$

$$= \frac{2(\sqrt{x^2 + xy})^2 - x(2x + y)}{4(x^2 + xy)\sqrt{x^2 + xy}} = \boxed{\frac{xy}{4(x^2 + xy)\sqrt{x^2 + xy}}}$$

$$z_{yy} = \frac{\partial_y(x)2\sqrt{x^2 + xy} - x\partial_y(2\sqrt{x^2 + xy})}{4(\sqrt{x^2 + xy})^2} = \frac{0 - x2\frac{x}{2\sqrt{x^2+xy}}}{4(x^2 + xy)}$$

$$= \boxed{-\frac{x^2}{4(x^2 + xy)\sqrt{x^2 + xy}}}$$

$$(3) z_x = \frac{\partial_x(x^2 + y^2)}{x^2 + y^2} = \boxed{\frac{2x}{x^2 + y^2}}, \quad z_y = \frac{\partial_y(x^2 + y^2)}{x^2 + y^2} = \boxed{\frac{2y}{x^2 + y^2}}$$

$$z_{xx} = \frac{\partial_x(2x)(x^2 + y^2) - 2x\partial_x(x^2 + y^2)}{(x^2 + y^2)^2} = \frac{2(x^2 + y^2) - 2x(2x)}{(x^2 + y^2)^2} = \boxed{\frac{2y^2 - 2x^2}{(x^2 + y^2)^2}}$$

$$z_{xy} = \frac{\partial_y(2x)(x^2 + y^2) - 2x\partial_y(x^2 + y^2)}{(x^2 + y^2)^2} = \frac{0 - 2x(2y)}{(x^2 + y^2)^2} = \boxed{-\frac{4xy}{(x^2 + y^2)^2}}$$

$$z_{yx} = \frac{\partial_x(2y)(x^2 + y^2) - 2y\partial_x(x^2 + y^2)}{(x^2 + y^2)^2} = \frac{0 - 2y(2x)}{(x^2 + y^2)^2} = \boxed{-\frac{4xy}{(x^2 + y^2)^2}}$$

$$z_{yy} = \frac{\partial_y(2y)(x^2 + y^2) - 2y\partial_y(x^2 + y^2)}{(x^2 + y^2)^2} = \frac{2(x^2 + y^2) - 2y(2y)}{(x^2 + y^2)^2} = \boxed{\frac{2x^2 - 2y^2}{(x^2 + y^2)^2}}$$

レポート 9 解答 No.2 18 点

(4) 注 z_x は公式 $(x^n)' = nx^{n-1}$ を、 z_y は公式 $(a^x)' = a^x \log a$ を使用

$$z_x = \boxed{yx^{y-1}}, \quad z_y = \boxed{x^y \log x}$$

$$z_{xx} = \boxed{y(y-1)x^{y-2}}$$

$$z_{xy} = \partial_y(y)x^{y-1} + y\partial_y(x^{y-1}) = x^{y-1} + yx^{y-1} \log x \partial_y(y-1) = \boxed{x^{y-1} + yx^{y-1} \log x}$$

$$z_{yx} = \partial_x(x^y) \log x + x^y \partial_x(\log x) = yx^{y-1} \log x + x^y \frac{1}{x} = \boxed{yx^{y-1} \log x + x^{y-1}}$$

$$z_{yy} = \partial_y(x^y) \log x = \boxed{x^y (\log x)^2}$$

(5) 注 $\partial_x \left(\frac{x}{y} \right) = \frac{1}{y}$, $\partial_y \left(\frac{x}{y} \right) = \partial_y(xy^{-1}) = x(-1)y^{-2} = -\frac{x}{y^2}$

$$z_x = \frac{1}{1 + \left(\frac{x}{y} \right)^2} \partial_x \left(\frac{x}{y} \right) = \frac{y^2}{x^2 + y^2} \left(\frac{1}{y} \right) = \boxed{\frac{y}{x^2 + y^2}}$$

$$z_y = \frac{1}{1 + \left(\frac{x}{y} \right)^2} \partial_y \left(\frac{x}{y} \right) = \frac{y^2}{x^2 + y^2} \left(-\frac{x}{y^2} \right) = \boxed{-\frac{x}{x^2 + y^2}}$$

$$z_{xx} = \frac{\partial_x(y)(x^2 + y^2) - y\partial_x(x^2 + y^2)}{(x^2 + y^2)^2} = \frac{0 - y(2x)}{(x^2 + y^2)^2} = \boxed{-\frac{2xy}{(x^2 + y^2)^2}}$$

$$z_{xy} = \frac{\partial_y(y)(x^2 + y^2) - y\partial_y(x^2 + y^2)}{(x^2 + y^2)^2} = \frac{(x^2 + y^2) - y(2y)}{(x^2 + y^2)^2} = \boxed{\frac{x^2 - y^2}{(x^2 + y^2)^2}}$$

$$z_{yx} = -\frac{\partial_x(x)(x^2 + y^2) - x\partial_x(x^2 + y^2)}{(x^2 + y^2)^2} = -\frac{(x^2 + y^2) - x(2x)}{(x^2 + y^2)^2} = \boxed{\frac{x^2 - y^2}{(x^2 + y^2)^2}}$$

$$z_{yy} = -\frac{\partial_y(x)(x^2 + y^2) - x\partial_y(x^2 + y^2)}{(x^2 + y^2)^2} = -\frac{0 - x(2y)}{(x^2 + y^2)^2} = \boxed{\frac{2xy}{(x^2 + y^2)^2}}$$

(6) $z_x = e^{2x+3y} \partial_x(2x+3y) = \boxed{2e^{2x+3y}}$, $z_y = e^{2x+3y} \partial_y(2x+3y) = \boxed{3e^{2x+3y}}$

$$z_{xx} = 2e^{2x+3y} \partial_x(2x+3y) = \boxed{4e^{2x+3y}}$$

$$z_{xy} = 2e^{2x+3y} \partial_y(2x+3y) = \boxed{6e^{2x+3y}}$$

$$z_{yx} = 3e^{2x+3y} \partial_x(2x+3y) = \boxed{6e^{2x+3y}}$$

$$z_{yy} = 3e^{2x+3y} \partial_y(2x+3y) = \boxed{9e^{2x+3y}}$$

レポート 11 解答 7 点

$$\begin{aligned}
 (1) \quad \iint_D x^2 y^3 dS &= \int_1^2 \int_{-1}^2 x^2 y^3 dy dx = \int_1^2 \left[x^2 \frac{y^4}{4} \right]_{-1}^2 dx = \int_1^2 x^2 \frac{2^4 - (-1)^4}{4} dx \\
 &= \int_1^2 \frac{15}{4} x^2 dx = \frac{15}{4} \left[\frac{x^3}{3} \right]_1^2 = \frac{15}{4} \frac{2^3 - 1^3}{3} = \boxed{\frac{35}{4}}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad \iint_D xy dS &= \int_0^1 \int_0^{x^2} xy dy dx = \int_0^1 \left[x \frac{y^2}{2} \right]_0^{x^2} dx = \int_0^1 x \frac{(x^2)^2 - 0}{2} dx \\
 &= \int_0^1 \frac{1}{2} x^5 dx = \frac{1}{2} \left[\frac{x^6}{6} \right]_0^1 = \boxed{\frac{1}{12}}
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad \iint_D \sqrt{x^2 - y^2} dS &= \int_0^1 \int_0^x \sqrt{x^2 - y^2} dy dx = \int_0^1 \left[\frac{1}{2} \left(y \sqrt{x^2 - y^2} + x^2 \sin^{-1} \frac{y}{x} \right) \right]_0^x dx \\
 &= \frac{1}{2} \int_0^1 (x\sqrt{0} + x^2 \sin^{-1} 1) - (0\sqrt{x^2} + x^2 \sin^{-1} 0) dx = \frac{1}{2} \int_0^1 \frac{\pi}{2} x^2 dx = \frac{\pi}{4} \left[\frac{x^3}{3} \right]_0^1 = \boxed{\frac{\pi}{12}}
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad \iint_D y dS &= \int_{-1}^2 \int_{y^2}^{y+2} y dx dy = \int_{-1}^2 [xy]_{y^2}^{y+2} dy = \int_{-1}^2 \{(y+2)y - y^2 y\} dy \\
 &= \int_{-1}^2 (-y^3 + y^2 + 2y) dy = \left[-\frac{y^4}{4} + \frac{y^3}{3} + 2\frac{y^2}{2} \right]_{-1}^2 \\
 &= \left(-\frac{2^4}{4} + \frac{2^3}{3} + 2^2 \right) - \left(-\frac{(-1)^4}{4} + \frac{(-1)^3}{3} + (-1)^2 \right) = \boxed{\frac{9}{4}}
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad \iint_D (x^2 + y^2) dS &= \int_0^1 \int_0^{1-x} (x^2 + y^2) dy dx = \int_0^1 \left[x^2 y + \frac{y^3}{3} \right]_0^{1-x} dy \\
 &= \int_0^1 \left\{ x^2(1-x) + \frac{(1-x)^3}{3} \right\} dy = \frac{1}{3} \int_0^1 \{ 3x^2 - 3x^3 + (1 - 3x + 3x^2 - x^3) \} dy \\
 &= \frac{1}{3} \int_0^1 (-4x^3 + 6x^2 - 3x + 1) dx = \frac{1}{3} \left[-x^4 + 2x^3 - \frac{3x^2}{2} + x \right]_0^1 = \frac{1}{3} \left(-1 + 2 - \frac{3}{2} + 1 \right) = \boxed{\frac{1}{6}}
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad \iint_D \cos(x+2y) dS &= \int_0^{\frac{\pi}{2}} \int_0^x \cos(x+2y) dy dx = \int_0^{\frac{\pi}{2}} \left[\frac{1}{2} \sin(x+2y) \right]_0^x dx \\
 &= \frac{1}{2} \int_0^{\frac{\pi}{2}} (\sin 3x - \sin x) dy = \frac{1}{2} \left[-\frac{1}{3} \cos 3x + \cos x \right]_0^{\frac{\pi}{2}} \\
 &= \frac{1}{2} \left\{ -\frac{1}{3} \cos \frac{3}{2}\pi + \cos \frac{1}{2}\pi - \left(-\frac{1}{3} \cos 0 + \cos 0 \right) \right\} = \frac{1}{2} \left(-0 + 0 + \frac{1}{3} - 1 \right) = \boxed{-\frac{1}{3}}
 \end{aligned}$$

$$\text{注 } \cos \frac{3}{2}\pi = 0, \quad \cos \frac{1}{2}\pi = 0, \quad \cos 0 = 1$$

$$\begin{aligned}
 (7) \quad \iint_D x dS &= \int_0^1 \int_{1-x}^{\sqrt{1-x^2}} x dy dx = \int_0^1 [xy]_{1-x}^{\sqrt{1-x^2}} dx = \int_0^1 \{ x\sqrt{1-x^2} - x(1-x) \} dx \\
 &= \left[-\frac{1}{3} \sqrt{1-x^2}^3 - \frac{x^2}{2} + \frac{x^3}{3} \right]_0^1 = -0 - \frac{1}{2} + \frac{1}{3} - \left(-\frac{1}{3} - 0 + 0 \right) = \boxed{\frac{1}{6}}
 \end{aligned}$$

$$\text{注 } \int x\sqrt{1-x^2} dx = \int xt^{\frac{1}{2}} \frac{1}{(-2x)} dt = -\frac{1}{3} t^{\frac{3}{2}} + C = -\frac{1}{3} \sqrt{1-x^2}^3 + C \quad (t = 1-x^2)$$

レポート 12 解答 8 点

$$(1) \frac{dy}{dx} = xy, \quad \frac{1}{y} dy = x dx, \quad \int \frac{1}{y} dy = \int x dx,$$

$$\log |y| = \frac{x^2}{2} + C, \quad y = \pm e^{\frac{x^2}{2} + C}, \quad \boxed{y = Ae^{\frac{x^2}{2}}} \quad \text{注} \quad A = \pm e^C$$

$$(2) y \frac{dy}{dx} = -x^2, \quad y dy = -x^2 dx, \quad \int y dy = - \int x^2 dx,$$

$$\frac{y^2}{2} = -\frac{x^3}{3} + C, \quad y^2 = -\frac{2}{3}x^3 + 2C, \quad \boxed{y = \pm \sqrt{C - \frac{2}{3}x^3}} \quad \text{注} \quad 2C \text{ を新たに } C \text{ とする}$$

$$(3) x^2 \frac{dy}{dx} = -y, \quad \frac{1}{y} dy = -\frac{1}{x^2} dx, \quad \int \frac{1}{y} dy = - \int x^{-2} dx,$$

$$\log |y| = \frac{1}{x} + C, \quad y = \pm e^{\frac{1}{x} + C}, \quad \boxed{y = Ae^{\frac{1}{x}}} \quad \text{注} \quad A = \pm e^C$$

$$(4) \frac{dy}{dx} = y^2 x, \quad \frac{1}{y^2} dy = x dx, \quad \int y^{-2} dy = \int x dx,$$

$$-\frac{1}{y} = \frac{x^2}{2} + C, \quad y = \frac{2}{-x^2 - 2C}, \quad \boxed{y = -\frac{2}{x^2 + C}} \quad \text{注} \quad C = 2C$$

$$(5) (x+1) \frac{dy}{dx} = y+1, \quad \frac{1}{y+1} dy = \frac{1}{x+1} dx, \quad \int \frac{1}{y+1} dy = \int \frac{1}{x+1} dx,$$

$$\log |y+1| = \log |x+1| + C, \quad \log \left| \frac{y+1}{x+1} \right| = C, \quad y+1 = A(x+1), \quad \boxed{y = A(x+1) - 1}$$

注 $A = \pm e^C$

$$(6) x^3 \frac{dy}{dx} = y^2, \quad \frac{1}{y^2} dy = \frac{1}{x^3} dx, \quad \int \frac{1}{y^2} dy = \int \frac{1}{x^3} dx,$$

$$-\frac{1}{y} = -\frac{1}{2x^2} + C, \quad y = \frac{2x^2}{1 - 2Cx^2}, \quad y = \frac{2x^2}{1 - Cx^2} \quad \text{注} \quad C = 2C$$

$$-1 = \frac{2 \cdot 1^2}{1 - C1^2}, \quad C - 1 = 2, \quad C = 3, \quad \boxed{y = \frac{2x^2}{1 - 3x^2}}$$

$$(7) x \frac{dy}{dx} = -y, \quad \frac{1}{y} dy = -\frac{1}{x} dx, \quad \int \frac{1}{y} dy = - \int \frac{1}{x} dx,$$

$$\log |y| = -\log |x| + C, \quad y = \pm e^{-\log |x| + C}, \quad y = A \frac{1}{|x|} = \frac{A}{x} \quad \text{注} \quad A = \pm A$$

$$1 = \frac{1}{A}, \quad A = 1, \quad \boxed{y = \frac{1}{x}}$$

$$(8) \frac{dy}{dx} = (y^2 + 4)x, \quad \frac{1}{y^2 + 4} dy = x dx, \quad \int \frac{1}{y^2 + 4} dy = \int x dx,$$

$$\frac{1}{2} \tan^{-1} \frac{y}{2} = \frac{x^2}{2} + C, \quad \tan^{-1} \frac{y}{2} = x^2 + 2C, \quad \frac{y}{2} = \tan(x^2 + 2C), \quad y = 2 \tan(x^2 + C)$$

注 $C = 2C$

$$2 = 2 \tan(0^2 + C), \quad 1 = \tan C, \quad C = \frac{\pi}{4}, \quad \boxed{y = 2 \tan \left(x^2 + \frac{\pi}{4} \right)}$$